Math 1B Quiz 8

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Name: ____ Score: /10

You have twenty minutes (plus the break) to complete the following closed-note open-chalkboard quiz. Partial credit will be awarded for correct work, and no points will be given for simply writing down the correct answer. Please box your final answers. Use the back of the page as necessary.

1. (5 pts) Determine whether the following series converges absolutely, converges conditionally, or diverges. Explain how you know.

$$\sum_{n=0}^{\infty} \frac{(-1)^n\, n^2}{2^n\, n!}$$

We use the ratio test:

$$\begin{aligned} \lim_{n\to\infty} \frac{(n+1)^2/(2^{n+1}(n+1)!)}{n^2/(2^n n!)} &= \lim_{n\to\infty} \frac{(n+1)^2}{n^2} \frac{2^n}{2^{n+1}} \frac{n!}{(n+1)!} \\ &= 1 \cdot \frac{1}{2} \cdot \lim_{n\to\infty} \frac{1}{n} \\ &= 0 < 1 \end{aligned}$$

Thus the series converges absolutely.

2. (0 pts) What is the next number in the following sequence?

$$4, 14, 23, 34, 42, 50, \ldots$$

The next number is 59. *These are the subway stops on the A-Line in Manhattan.*

3. (5 pts) Determine whether the following series converges absolutely, converges conditionally, or diverges. Explain how you know.

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n+e^{-n}}$$

Since $n + e^{-n} \approx n$, *and* $\sum(-1)^n/n$ *converges conditionally, we expect the answer to be "conditional convergence". Indeed, we have the alternating series test:* $(n + 1) + e^{-(n+1)} \geq n + e^n$ *for all* n, *so* $1/(n + e^{-n})$ *is decreasing, and is less than* $1/n$, *so tends to* 0. *Hence the alternating sum* $\sum(-1)^n/(n + e^{-n})$ *converges. But* $n+e^{-n} \leq n+1$, *and* $\sum 1/(n+1)$ *diverges by the* p*-test with* $p = 1$. *So* $\sum 1/(n+e^{-n}) \geq \sum 1/(n + 1) = \infty$, *so the series does not converge absolutely. Hence the series converges conditionally.*