

Math 1B Quiz 3

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<http://math.berkeley.edu/~theo/f/08Summer1B/>

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Name: _____ Score: _____ /10

You have twenty minutes to complete the following quiz. The quiz is closed-note but open-chalkboard. Although you do not need to write down every step of your calculation, you do need to show enough work that I know how you did each problem (no points will be given for simply writing down the correct answer). Partial credit will be awarded for correct work. Please box your final answers. You do not need to simplify your final answers.

1. (5 pts) Determine whether the following integral converges or diverges. If it converges, evaluate it to find a number. If it diverges, explain how you know.

$$\int_0^{\infty} \frac{x}{(x^2 + 3)^2} dx$$

The only possible divergence is at $x \approx \infty$, where the power is $x/x^4 = x^{-3}$. Since $\int_1^{\infty} x^{-3} dx$ converges, we expect this integral to converge as well. Indeed, substituting $u = x^2 + 3$, we have

$$\begin{aligned} \int_0^{\infty} \frac{x}{(x^2 + 3)^2} dx &= \int_3^{\infty} \frac{du/2}{u^2} \\ &= \left. \frac{-1}{2} \frac{1}{u} \right]_3^{\infty} \\ &= \frac{1}{6} - \frac{1}{\infty} = \boxed{\frac{1}{6}} \end{aligned}$$

2. (0 pts) What is one thing you like about this class so far? What is one thing you don't like?

I like all the awesome and engaged students. I don't like that we have to rush through the material.

3. (5 pts) Determine whether the following integral converges or diverges. If it converges, evaluate it to find a number. If it diverges, explain how you know.

$$\int_0^1 \frac{5}{x^6} dx$$

The integral diverges. In general, any integral near 0 of $1/x^p$ diverges for $p \geq 1$.