

Math 1A: Discussion Exercises

GSI: Theo Johnson-Freyd

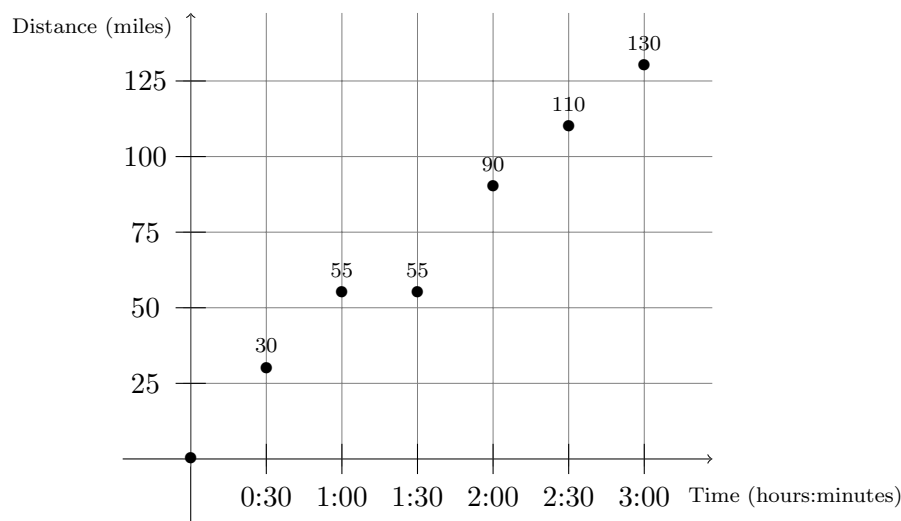
<http://math.berkeley.edu/~theo/jf/09Spring1A/>

Find two or three classmates and a few feet of chalkboard. Introduce yourself to your new friends, and write all of your names at the top of the chalkboard. As a group, try your hand at the following exercises. Be sure to discuss how to solve the exercises — *how* you get the solution is much more important than *whether* you get the solution. If as a group you agree that you all understand a certain type of exercise, move on to later problems. You are not expected to solve all the exercises: in particular, the last few exercises may be very hard.

Many of the exercises are from *Single Variable Calculus: Early Transcendentals for UC Berkeley* by James Stewart; these are marked with an §. Others my own, or are independently marked.

Preview and Review

1. A driver wrote down the total distance she had traveled each half-hour during her three-hour trip, and plotted the points on a graph:



- (a) What other ways could she have presented the same data?
- (b) Does the above graph define Distance as a function of Time? Why or why not? If so, what is the domain and range? Is the function monotone?
- (c) Does the above graph define Time as a function of Distance? Why or why not? If so, what is the domain and range? Is the function monotone?
- (d) What is the driver's average speed during each hour of her trip? What does the average speed correspond to on the graph?
- (e) Another driver also takes a three-hour trip. His average speeds over each half hour were (mph):

40, 50, 40, 60, 20, 60

How far had this driver gone after each half hour?

2. § Sketch a rough graph of the outdoor temperature as a function of time during a typical spring day.

3. § A homeowner mows the lawn every Wednesday afternoon. Sketch a rough graph of the height of the grass as a function of time over the course of a four-week period.
4. § A spherical balloon with radius r inches has volume $V(r) = \frac{4}{3}\pi r^3$. Find a function that represents the amount of air required to inflate the balloon from a radius of r inches to a radius of $r + 1$ inches.
5. § For each of the following functions, evaluate the corresponding *difference quotient*:
 - (a) $f(x) = 4 + 3x - x^2$, $\frac{f(3+h) - f(3)}{h}$
 - (b) $f(x) = x^3$, $\frac{f(a+h) - f(a)}{h}$
 - (c) $f(x) = \frac{1}{x}$, $\frac{f(x) - f(a)}{x - a}$
6. § Find the domain of the following functions (assume that the domain is as large as possible, provided that the codomain is \mathbb{R} , the set of all real numbers):
 - (a) $f(x) = \frac{x}{3x-1}$
 - (b) $f(t) = \sqrt{t} + \sqrt[3]{t}$
 - (c) $g(u) = \sqrt{u} + \sqrt{4-u}$
 - (d) $h(x) = \frac{1}{\sqrt[4]{x^2-5x}}$
7. § Sketch the graph $\{y = f(x)\}$ of the following functions:
 - (a) $f(x) = 5$
 - (b) $f(x) = \frac{1}{2}(x+3)$
 - (c) $f(x) = \frac{x}{|x|}$
 - (d) $f(x) = \frac{3x+|x|}{x}$
 - (e) $f(x) = \frac{|x|}{x^2}$
 - (f) $f(x) = \begin{cases} x+9 & \text{if } x < -3 \\ -2x & \text{if } |x| \leq 3 \\ -6 & \text{if } x > 3 \end{cases}$
8. § Find an expression for the function whose graph is the line segment joining the points $(1, -3)$ and $(5, 7)$. Be very precise (hint: specify the domain of your function).
9. § A rectangle has perimeter 20 m. Express the area of the rectangle as a function of the length of one of its sides. Be sure to use units. What is the domain of your function?
10. § Determine whether each function is even, odd, or neither.
 - (a) $f(x) = \frac{x}{x^2+1}$
 - (b) $f(x) = \frac{x^2}{x^4+1}$
 - (c) $f(x) = 1 + 3x^2 - x^4$
 - (d) $f(x) = \frac{x}{x+1}$
 - (e) $f(x) = 1 + 3x^3 - x^5$
 - (f) $f(x) = x|x|$
11. § In a certain country, income tax is assessed as follows: There is no tax on income up to \$10 000; any income over \$10 000 is taxed at a rate of 10%, up to an income of \$20 000; any income over \$20 000 is taxed at 15%.

Sketch a graph of the tax rate as a function of income. Sketch a graph of the total assessed tax as a function of income.