Math 1A: Discussion Exercises

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Find two or three classmates and a few feet of chalkboard. Introduce yourself to your new friends, and write all of your names at the top of the chalkboard. As a group, try your hand at the following exercises. Be sure to discuss how to solve the exercises — *how* you get the solution is much more important than *whether* you get the solution. If as a group you agree that you all understand a certain type of exercise, move on to later problems. You are not expected to solve all the exercises: in particular, the last few exercises may be very hard.

Many of the exercises are from *Single Variable Calculus: Early Transcendentals for UC Berkeley* by James Stewart; these are marked with an §. Others are my own, or are independently marked.

More Preview and Review

1. § Classify each function as a power function, root function, polynomial (state its degree), rational function, algebraic function, trigonometric function, exponential functions, or logarithmic function. Some functions are more than one of these, and some are none.

   (a) $\sqrt[5]{x}$ (c) $\tan 2x$ (e) $\sqrt{1-x^2}$ (g) $\log_{10} x$

   (b) $x^9 + x^4$ (d) $(\sin x)^2$ (f) $\dfrac{x^2+1}{x^3+x}$ (h) $\ln \sin x$

   Remark: We will develop powerful techniques with which to manipulate (in particular, differentiate and integrate) all these functions. Exactly what techniques we will use will depend on the type of function. For example, the *derivative* (we haven't defined this yet) of a power function is

   $$\mathcal{D}\left[x^a\right] = a \times x^{a-1} \tag{1}$$

   whereas the derivative of an exponential function is

   $$\mathcal{D}\left[a^x\right] = \ln a \times a^x. \tag{2}$$

   One of the most common mistakes in Math 1A is to use the formula from equation (1) when (2) is correct. The best way to avoid the mistake is to get very good at telling what type of function you're dealing with, and understanding how the different types are different.

2. (a) § What do all members of the family of linear functions $f(x) = 1 + m(x+3)$ have in common? Sketch several members of the family.

   (b) § What do all members of the family of linear functions $f(x) = c - x$ have in common? Sketch several members of the family.

   (c) Which functions (if any) belong to both families?

3. § If the recommended adult dosage for a drug is D (in mg), then to determine the appropriate dosage c for a child of age a, pharmacists use the equation $c(a) = 0.0417D(a+1)$.

   Remark: c is a function of both a and D, but for the purposes of this problem we're thinking of a as the independent variable and c as the dependent variable, so that c is a function of a, and we're thinking of D as a "fixed unknown constant", which is actually just another word for "variable".

(a) What does the slope of the graph of $c(a)$ represent? If the dosage for an adult for this particular drug is 200 mg, what is this slope?

(b) What is the dosage for a newborn in terms of D? What feature of the graph of $c(a)$ corresponds to this amount?

(c) What is the domain of the function $c(a)$? This is a rather tricky question: think about the real-word set-up. For instance, is $a = 100$ in the domain? What about $a = \pi$?

4. Kepler's Third Law of Planetary Motion states that

   > The square of the period of revolution of a planet is proportional to the cube of its mean distance from the sun

   Let's use units so that we measure mean distance in Astronomical Units (AU), defined so that the mean distance from the Earth to the Sun is 1 AU (1 AU is 149 598 000 kilometers), and so that we measure periods of revolution in years, defined so that the period of revolution of the Earth is 1 year.

   (a) Choose variables to represent all quantities in the above Third Law. Rewrite the Third Law as an equation.

   (b) Solve this equation to express the mean distance as a function of the period of revolution, and also solve it to express the period of revolution as a function of the mean distance. What kinds of functions are these?

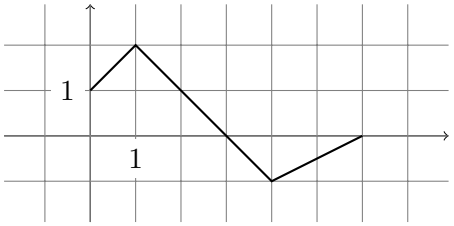
   (c) Mars is roughly 1.5 AU from the Sun. What is its period of revolution?
   Saturn's period of revolution is just about 30 years. What is its mean distance?

5. § Suppose you are given the graph of $y = f(x)$. Write equations (in terms of f) for the functions whose graphs are obtained from f as follows:

   (a) Shifted upward by 3.
   (b) Shifted leftward by 3.
   (c) Shifted rightward by 3.
   (d) Reflected about the x-axis.
   (e) Reflected about the y-axis.
   (f) Reflected about the origin.
   (g) Stretched vertically by a factor of 3.
   (h) Stretched horizontally by a factor of 3.
   (i) Stretched in both directions by a factor of 3.
   (j) Shrunk vertically by a factor of 3.
   (k) Shrunk horizontally by a factor of 3.
   (l) Shrunk in both directions by a factor of 3.

6. § To the right is the graph of $y = f(x)$. Use it to graph the following functions:

   (a) $y = f(2x)$
   (b) $y = f(x + 2)$
   (c) $y = f(-x)$
   (d) $y = -f(-x) + 1$



7. § How is the graph of $y = f(|x|)$ related to the graph of $y = f(x)$? Sketch the graphs of $y = \sin|x|$, $y = \sqrt{|x|}$, and (most importantly for our class) $y = \ln|x|$.

8. § Suppose that f and g are even functions. What can you say about $f + g$ and fg? What if f and g are both odd?

9. Use algebra to show the shifting a graph by a units upward and then stretching vertically by a factor of b is the same as first stretching the graph vertically by a factor of b and then shifting upward by ab.