

Math 1A: Discussion Exercises

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<http://math.berkeley.edu/~theo/f/09Spring1A/>

Find two or three classmates and a few feet of chalkboard. As a group, try your hand at the following exercises. Be sure to discuss how to solve the exercises — *how* you get the solution is much more important than *whether* you get the solution. If as a group you agree that you all understand a certain type of exercise, move on to later problems. You are not expected to solve all the exercises: in particular, the last few exercises may be very hard.

Many of the exercises are from *Single Variable Calculus: Early Transcendentals for UC Berkeley* by James Stewart; these are marked with an §. Others are my own, or are independently marked.

Introducing Limits

1. Let $\lfloor x \rfloor$ be the *floor* of x , i.e. the greatest integer that is less than or equal to x .
 - (a) Sketch the graph of the function $y = \lfloor x \rfloor$.
 - (b) For what a does $\lim_{x \rightarrow a} \lfloor x \rfloor$ exist? Why?
2. Let $\delta_{\mathbb{Z}}(x)$ be the function that is 1 if x is an integer and 0 if x is not an integer.
 - (a) Sketch the graph of the function $y = \delta_{\mathbb{Z}}(x)$.
 - (b) For what a does $\lim_{x \rightarrow a} \delta_{\mathbb{Z}}(x)$ exist? Why?
3. Show that $\lfloor x \rfloor + \lfloor -x \rfloor = \delta_{\mathbb{Z}}(x) - 1$. What does this say about the Sum and Difference limit laws?
4. § Sketch the graph of an example of a function f such that $\lim_{x \rightarrow 3^+} f(x) = 4$, $\lim_{x \rightarrow 3^-} f(x) = 2$, $\lim_{x \rightarrow -2} f(x) = 2$, $f(3) = 3$, $f(-2) = 1$, and $\lim_{x \rightarrow 1} f(x) = +\infty$.
5. § Determine the value of $\lim_{x \rightarrow -3^+} \frac{x+2}{x+3}$. *Hint:* Draw a picture.
6. Determine the value of $\lim_{x \rightarrow 2^-} \frac{x^2 - 3x + 2}{x^2 + x - 6}$. *Hint:* Factor.
7. § In the theory of Special Relativity, the mass of a particle with velocity v is

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

where m_0 is the mass of the particle at rest and c is the speed of light.

- (a) What happens as $v \rightarrow c^-$?
 - (b) Does it make sense to ask about $v \rightarrow c^+$?
8. § Evaluate $\lim_{x \rightarrow 0} \frac{|2x-1| - |2x+1|}{x}$.
 9. § If $\lim_{x \rightarrow a} [f(x) + g(x)] = 2$ and $\lim_{x \rightarrow a} [f(x) - g(x)] = 1$, find $\lim_{x \rightarrow a} [f(x)g(x)]$. *Warning:* $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ might not exist.
 10. True or False:

$$\text{If } \lim_{x \rightarrow 1} [f(x)]^2 = 4, \text{ then } \lim_{x \rightarrow 1} f(x) = 2$$

If true, prove it. If false, find a counterexample.