Math 1A: Discussion Exercises

GSI: Theo Johnson-Freyd
http://math.berkeley.edu/~theojf/09Spring1A/

Find two or three classmates and a few feet of chalkboard. As a group, try your hand at the following exercises. Be sure to discuss how to solve the exercises — *how* you get the solution is much more important than *whether* you get the solution. If as a group you agree that you all understand a certain type of exercise, move on to later problems. You are not expected to solve all the exercises: in particular, the last few exercises may be very hard.

Many of the exercises are from *Single Variable Calculus: Early Transcendentals for UC Berkeley* by James Stewart; these are marked with an §. Others are my own, or are independently marked.

Introducing derivatives

1. § Sketch the graph of a function f for which $f(0) = 0$, $f'(0) = 3$, $f'(1) = 0$, and $f'(2) = -1$.

2. Sketch the graph of the derivative of the following function:
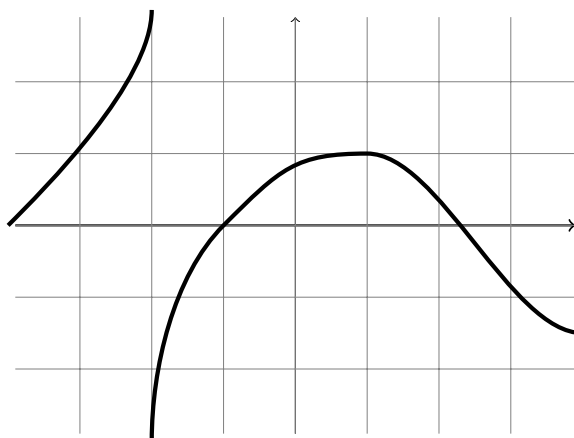
3. § Each of the following limits represents a derivative of some function f at some number a. State such an f and a in each case. Then use known Derivative Laws (only use laws allowed on next week's midterm: power, sum, difference, and multiplication by a constant) to compute the corresponding derivative.

   (a) $\lim\limits_{h \to 0} \dfrac{(1+h)^{10} - 1}{h}$ (b) $\lim\limits_{h \to 0} \dfrac{\sqrt[4]{16+h} - 2}{h}$ (c) $\lim\limits_{x \to 5} \dfrac{3x^2 - 75}{x - 5}$ (d) $\lim\limits_{t \to 1} \dfrac{t^4 + t - 2}{t - 1}$

4. § For each of the following functions, find $f'(a)$. You may use only: the definition of the derivative, and Derivative Laws allowed on next week's midterm.

   (a) $f(t) = \dfrac{2t+1}{t+3}$ (b) $f(x) = \dfrac{1}{\sqrt{x+2}}$ (c) $\sqrt{3x+1}$ (d) $f(x) = 3 - 2x + 4x^2$

5. § Make a careful sketch of the graph of $y = \sin x$, and sketch the graph of the derivative $\sin' x$. In particular, what are the zeros of $\sin'$, and where is it positive and negative. Can you guess the formula for $\sin'$ based on the graph?

6. What is the domain of the function $f(x) = \sqrt{x}$? What is the domain of its derivative $f'(x)$?

7. Find an equation for the line tangent to $y = x^3 - x^2$ at $(2, 4)$. Find all points where the tangent line has slope 2. Find all tangent lines to the curve that pass through $(0, 1)$.