# Math 1A: Discussion Exercises

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Find two or three classmates and a few feet of chalkboard. As a group, try your hand at the following exercises. Be sure to discuss how to solve the exercises — *how* you get the solution is much more important than *whether* you get the solution. If as a group you agree that you all understand a certain type of exercise, move on to later problems. You are not expected to solve all the exercises: in particular, the last few exercises may be very hard.

Many of the exercises are from *Single Variable Calculus: Early Transcendentals for UC Berkeley* by James Stewart; these are marked with an §. Others are my own, or are independently marked.

## Product and Quotient Rules

1. § Let $f(5) = 1$, $f'(5) = 6$, $g(5) = -3$, and $g'(5) = 2$. Find $(fg)'(5)$, $(f/g)'(5)$, and $(g/f)'(5)$.
2. § Differentiate.
   (a) $(x^3 + 2x)e^x$
   (b) $\dfrac{x+1}{x^3+x-2}$
   (c) $(u^{-2} + u^{-3})(u^5 - 2u^2)$
   (d) $\dfrac{t}{(t-1)^2}$
   (e) $\dfrac{2t}{4+t^2}$
   (f) $\dfrac{ax+b}{cx+d}$
3. § Find $f'(x)$ and $f''(x)$:
   (a) $f(x) = x^4 e^x$
   (b) $f(x) = x^{5/2} e^x$
   (c) $f(x) = \dfrac{x^2}{1+2x}$
4. § How many tangent lines to the curve $y = x/(x+1)$ pass through the point $(1, 2)$? At what points do these tangent lines touch the curve?
5. § Use the Product Rule twice to prove that if $f, g, h$ are differentiable, then $(fgh)' = f'gh + fg'h + fgh'$. Then take $f = g = h$ to show that $\frac{d}{dx}\big[f(x)\big]^3 = 3\big[f(x)\big]^2 f'(x)$, and use this to differentiate $y = e^{3x}$.
6. § If $f$ and $g$ are differentiable, show that $(fg)'' = f''g + 2f'g' + fg''$. Find similar formulas for $(fg)'''$ and $(fg)^{(4)}$. Do you notice a pattern? Guess a formula for $(fg)^{(n)}$.
7. (a) Define $\exp_a(x) = a^x$ for $a$ a positive constant. Use the definition of derivative to prove that $\exp_a'(x) = \exp_a(x)\, \exp_a'(0)$.
   (b) Use the product rule and exponentiation laws to show that $\exp_{ab}'(0) = \exp_a'(0) + \exp_b'(0)$ if $a, b > 0$. What does the quotient rule say about $\exp_{a/b}'(0)$?
   (c) What is the function $\ell(a) = \exp_a'(0)$ as a function of $a$? (Hint: what is $\ell(e)$?)
8. Write out the first few derivatives $(f, f', f'', \dots)$ of $f(x) = xe^x$. Do you notice a pattern?
9. (a) Prove that if $p$ is a polynomial of degree $n$, then the derivative of $p(x)\, e^x$ is $q(x)\, e^x$, where $q$ is also a polynomial of degree $n$.
   (b) Let $f(x) = p(x)\, e^x$ where $p$ is a polynomial. What is $\lim_{x\to-\infty} f^{(n)}(x)$?
10. (a) **Really Hard.** Use the product rule to prove the quotient rule.
    (b) **Really Hard.** Let $p$ be a polynomial. Use the product rule, but not the chain rule, to prove that $\frac{d}{dx}\big[p(q(x))\big] = p'(q(x))\, q'(x)$.