Math 1A: Discussion Exercises

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Find two or three classmates and a few feet of chalkboard. As a group, try your hand at the following exercises. Be sure to discuss how to solve the exercises — *how* you get the solution is much more important than *whether* you get the solution. If as a group you agree that you all understand a certain type of exercise, move on to later problems. You are not expected to solve all the exercises: in particular, the last few exercises may be very hard.

Many of the exercises are from *Single Variable Calculus: Early Transcendentals for UC Berkeley* by James Stewart; these are marked with an §. Others are my own, or are independently marked.

Implicit Differentiation

1. § Find dy/dx by implicit differentiation.
   (a) $2x^3 + x^2y - xy^3 = 2$
   (b) $1 + x = \sin(xy^2)$
   (c) $y\sin(x^2) = x\sin(y^2)$
   (d) $\sqrt{x+y} = 1 + x^2y^2$
2. § Differentiate.
   (a) $\arctan\sqrt{x}$
   (b) $\arcsin(2x+1)$
   (c) $\sqrt{1-x^2}\arccos x$
   (d) $\operatorname{arccot} t + \operatorname{arccot}(1/t)$
   (e) $\arccos(e^{2x})$
   (f) $\arcsin\sqrt{\sin\theta}$
3. § If $f(x) + x^2[f(x)]^3 = 10$ and $f(1) = 2$, find $f'(1)$.
4. (a) § Find an equation of the tangent line to the curve $x^2 + y^2 = (2x^2 + 2y^2 - x)^2$ at the point $(0, -\frac{1}{2})$.
   (b) Find all points on the curve where the tangent line is horizontal.
5. § Find y'' by implicit differentiation.
   (a) $\sqrt{x} + \sqrt{y} = 1$
   (b) $x^4 + y^4 = a^4$
6. § Show that the sum of the x- and y- intercepts of any tangent line to the curve $\sqrt{x}+\sqrt{y} = \sqrt{c}$ is equal to c.
7. § Show that every curve in the family $y = ax^3$ is orthogonal to every curve in the family $x^2 + 3y^2 = b$, where a and b range over all real numbers.

Questions on earlier material

8. What's the derivative of $\sin^2 x$? What's the derivative of $\cos^2 x$? What happens when you add them together and why?
9. Find numbers A and B so that
$$\frac{d}{dx}\left[Ae^x\cos x + Be^x\sin x\right] = e^x\cos x$$
10. Find numbers α and β so that $y = e^{\alpha x}\sin(\beta x)$ is a solution to the differential equation:
$$y'' + 4y' + 5y = 0$$
Check that $y = e^{\alpha x}\cos(\beta x)$ is also a solution.