Math 1A: Discussion Exercises

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Find two or three classmates and a few feet of chalkboard. As a group, try your hand at the following exercises. Be sure to discuss how to solve the exercises — *how* you get the solution is much more important than *whether* you get the solution. If as a group you agree that you all understand a certain type of exercise, move on to later problems. You are not expected to solve all the exercises: in particular, the last few exercises may be very hard.

Many of the exercises are from *Single Variable Calculus: Early Transcendentals for UC Berkeley* by James Stewart; these are marked with an §. Others are my own, or are independently marked.

Linear Approximation and Differentials

1. § Verify the following linear approximations, for $x \approx 0$:
   (a) $\sqrt[3]{1-x} \approx 1 - \frac{1}{3}x$ (b) $1/(1+2x)^4 \approx 1 - 8x$ (c) $\sinh x \approx x$
2. § Find the linearization $L(x)$ of the function $f(x)$ at $x = a$.
   (a) $f(x) = x^4 + 3x^2$, $a = -1$ (b) $f(x) = \ln x$, $a = 1$ (c) $f(x) = \cos x$, $a = \pi/2$
3. § Find dy:
   (a) $y = x^2 \sin 2x$ (c) $y = (u+1)/(u-1)$ (e) $y = e^{-u}\cos u$
   (b) $y = s/(1+2s)$ (d) $y = \ln\sqrt{1+t^2}$ (f) $(1+r^3)^{-2}$
4. § For each of the following functions, (a) find the differential dy, (b) evaluate for the given values of x and dx, and (c) use the differential to approximation the value $y(x+dx)$.
   (a) $y = e^{x/10}$, $x = 0$, $dx = 0.1$ (c) $y = \tan x$, $x = \pi/4$, $dx = -0.1$
   (b) $y = e^{\tan x}$, $x = 0$, $dx = 0.1$ (d) $y = \sqrt{1+\ln x}$, $x = 0$, $dx = 0.1$
5. § Without a calculator, estimate the values of the following numbers:
   (a) $(2.001)^5$ (c) $(8.06)^{2/3}$ (e) $\sqrt{99.8}$
   (b) $e^{-0.015}$ (d) $\tan 44^\circ$ (f) $\sec 0.08$
6. § The circumference of a sphere is measured to be 84 cm, with a possible error of 0.5 cm. Use differentials to estimate the maximum error in the calculated surface area. What is the relative error? Use differentials to estimate the maximum error in the calculated volume. What is the relative error?
7. If y is a function of two variables, then dy depends on those variable and their differentials, so it depends on four numbers. Find dy when $y = r\sin\theta$.
8. § When blood flows along a blood vessel, Poiseuille's Law says that the flux F (the volume of blood per unit time that flows past a given point) is proportional to the fourth power of the radius R of the blood vessel:
   $$F = kR^4$$
   Show that the relative change in F is roughly four times the relative change in R. What happens if there is a clog that decreases the radius by 5%?
9. Use the example of compounded interest to illustrate the limits of linear approximation.