Math 1A: Discussion Exercises

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Find two or three classmates and a few feet of chalkboard. As a group, try your hand at the following exercises. Be sure to discuss how to solve the exercises — *how* you get the solution is much more important than *whether* you get the solution. If as a group you agree that you all understand a certain type of exercise, move on to later problems. You are not expected to solve all the exercises: in particular, the last few exercises may be very hard.

Many of the exercises are from *Single Variable Calculus: Early Transcendentals for UC Berkeley* by James Stewart; these are marked with an §. Others are my own, or are independently marked.

L'Hospital's Rule

1. § Find the limit. Use l'Hospital's Rule where appropriate, but also look for other methods.

   (a) $\lim_{x\to\infty} \frac{e^x}{x^3}$

   (b) $\lim_{x\to 1} \frac{\ln x}{\sin \pi x}$

   (c) $\lim_{x\to 0} \frac{\tanh x}{\tan x}$

   (d) $\lim_{t\to 0} \frac{5^t - 3^t}{t}$

   (e) $\lim_{x\to 0} \frac{\arcsin x}{x}$

   (f) $\lim_{x\to 0} \frac{1-\cos x}{x^2}$

   (g) $\lim_{x\to 0} \frac{x+\sin x}{x+\cos x}$

   (h) $\lim_{x\to\infty} \frac{\sqrt{x^2+2}}{\sqrt{2x^2+1}}$

   (i) $\lim_{x\to 1} \frac{x^a - ax + a - 1}{(x-1)^2}$

   (j) $\lim_{x\to a^+} \frac{\cos x \ln(x-a)}{\ln(e^x - e^a)}$

   (k) $\lim_{x\to\infty} \big(x \sin(\pi/x)\big)$

   (l) $\lim_{x\to 0} \big(\cot 2x \sin 6x\big)$

   (m) $\lim_{x\to\infty} \big(x^3 e^{-x^2}\big)$

   (n) $\lim_{x\to 1} \left(\frac{x}{x-1} - \frac{1}{\ln x}\right)$

   (o) $\lim_{x\to\infty} (x - \ln x)$

   (p) $\lim_{x\to\infty} \left(\sqrt{x^2+x} - x\right)$

   (q) $\lim_{x\to\infty} \left(xe^{1/x} - x\right)$

   (r) $\lim_{x\to\infty} \left(1 + \frac{a}{x}\right)^{bx}$

   (s) $\lim_{x\to\infty} \left(1 + \frac{3}{x} + \frac{5}{x^2}\right)^x$

   (t) $\lim_{x\to 0^+} (\cos x)^{1/x^2}$

   (u) $\lim_{x\to 0} (1-2x)^{1/x}$

2. § Prove that for any number $p > 0$: $\lim_{x\to\infty} \frac{e^x}{x^p} = \infty$ and $\lim_{x\to\infty} \frac{\ln x}{x^p} = 0$

3. § Try using l'Hospital's Rule to evaluate $\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}}$, and then evaluate the limit using some other method.

4. § L'Hospital's Rule was first published in *Analyse des Infiment Petits* in 1696 — this was the first calculus textbook ever. Solve the example used in that book ($a > 0$):

$$\lim_{x\to a} \frac{\sqrt{2a^3x - x^4} - a\sqrt[3]{a^2x}}{a - \sqrt[4]{ax^3}}$$

5. § Find the values of a and b so that $\lim_{x\to 0} \left(\frac{\sin 2x}{x^3} + a + \frac{b}{x^2}\right) = 0$.

6. § If f'' is continuous, prove: $\lim_{h\to 0} \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} = 0$.