Math 1A: Discussion Exercises

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Find two or three classmates and a few feet of chalkboard. As a group, try your hand at the following exercises. Be sure to discuss how to solve the exercises — *how* you get the solution is much more important than *whether* you get the solution. If as a group you agree that you all understand a certain type of exercise, move on to later problems. You are not expected to solve all the exercises: in particular, the last few exercises may be very hard.

Many of the exercises are from *Single Variable Calculus: Early Transcendentals for UC Berkeley* by James Stewart; these are marked with an §. Others are my own, or are independently marked.

Newton's Method

1. Recall that in Newton's Method, we approximate roots of functions by iterating $x_{n+1} = x_n - f(x_n)/f'(x_n)$. § Use Newton's Method to find x_3 for the specified $f(x)$ and x_1:

   (a) $f(x) = x^3 + 2x - 4$, $x_1 = 1$ (b) $f(x) = x^5 - x - 1$, $x_1 = 1$

2. § Use Newton's Method to find $\sqrt[5]{20}$, correct to three decimal places. Hint: what easy equation is $\sqrt[5]{20}$ a root of?

3. § Use Newton's Method to find the positive root of $x^2 = \sin x$, correct to three decimal places.

4. § Use Newton's Method to find all roots of $e^x = 3 - 2x$, correct to three decimal places.

5. § Apply Newton's Method to the equation $x^2 - a = 0$ to derive an iterative formula for computing $\sqrt{a}$.

6. § Apply Newton's Method to the equation $a - 1/x = 0$ to derive an iterative formula for computing $1/a$.

7. What happens when you apply Newton's Method to the function $f(x) = \sqrt[3]{x}$?

8. What happens when you apply Newton's Method to the function $f(x) = x^3 - 2x + 2$ with starting location $x_1 = 0$?

9. § Use Newton's Method to find the absolute maximum of the function $f(x) = x \cos x$ in $0 \le x \le \pi$, correct to three decimal places.

There is much deep mathematics related to Newton's Method, largely beyond the scope of this class. See for example `http://en.wikipedia.org/wiki/Newton_fractal` and related pages for an explanation of how Newton's Method is connected to fractals like the one behind this page. (Image from `http://www.cattail.nu/photoshop/brushes/classic_fractal.html`. A particularly nice gallery of "Newton Fractals" is at `http://local.wasp.uwa.edu.au/~pbourke/fractals/newtonraphson/`.)