Math 1A: Discussion Exercises

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Find two or three classmates and a few feet of chalkboard. As a group, try your hand at the following exercises. Be sure to discuss how to solve the exercises — *how* you get the solution is much more important than *whether* you get the solution. If as a group you agree that you all understand a certain type of exercise, move on to later problems. You are not expected to solve all the exercises: in particular, the last few exercises may be very hard.

Many of the exercises are from *Single Variable Calculus: Early Transcendentals for UC Berkeley* by James Stewart; these are marked with an §. Others are my own, or are independently marked.

Antiderivatives

1. Let F be an antiderivative of f and G an antiderivative of g. Decide whether each of the following statements is (always) TRUE or (sometimes) FALSE? Why or why not? If the statement is true, prove it. If the statement is false, give a counterexample.
   (a) The sum $F+G$ is an antiderivative of $f+g$.
   (b) The difference $F-G$ is an antiderivative of $f-g$.
   (c) The product FG is an antiderivative of fg.
   (d) The quotient F/G is an antiderivative of f/g.
   (e) The composition $F \circ G$ is an antiderivative of $f \circ g$.
   (f) If c is a constant, then $F+c$ is an antiderivative of $f+c$.
   (g) If c is a constant, then $F+c$ is an antiderivaitve of f.
   (h) If c is a constant, then cF is an antiderivative of cf.
   (i) If c is a constant, then cF is an antiderivative of f.
   (j) If $f=g$, then $F=G$.
   (k) If $F=G$, then $f=g$.
   (l) If $f=g$, then $F=G+c$ for some constant c.
   (m) If $f=g$ on an interval, then for some constant c, $F=G+c$ on that interval.
   (n) If $fG+gF=0$, then FG is locally a constant (meaning that it is constant on any interval for which f and g are both defined).
   (o) If $fG-gF=0$, then F/G is locally a constant.

2. § Find the most general antiderivative of the function:
   (a) $f(x)=\frac{1}{2}x^2-2x+6$
   (b) $f(x)=2x+3x^{1.7}$
   (c) $f(x)=(x+1)(2x-1)$
   (d) $f(x)=3e^x+7\sec^2 x$
   (e) $f(x)=\dfrac{x^5-x^3+2x}{x^4}$

3. § Find the most general function f such that:
   (a) $f''(x)=2+x^3+x^6$
   (b) $f'''(t)=t-\sqrt{t}$

4. § Does there exist a function f such that $f'(x)=x^{-1/3}$, $f(1)=1$, and $f(-1)=-1$?

5. § Solve the differential equation (i.e. find f given the data):

(a) $f'(x) = 8x^3 + 12x + 3,\ f(1) = 6$

(b) $f'(x) = \sqrt{x}(6 + 5x),\ f(1) = 10$

(c) $f'(t) = 2\cos t + \sec^2 t,\ f(\pi/3) = 4,$ $-\pi/2 < t < \pi/2$

(d) $f'(x) = 4/\sqrt{1 - x^2},\ f(\frac{1}{2}) = 1$

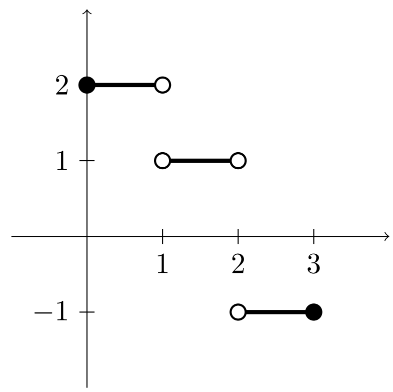
(e) $f''(x) = 4 - 6x - 40x^3,\ f(0) = 2,\ f'(0) = 1$

(f) $f''(\theta) = \sin\theta + \cos\theta,\ f(0) = 3,\ f'(0) = 4$

(g) $f''(x) = 20x^3 + 12x^2 + 4,\ f(0) = 8,\ f(1) = 5$

(h) $f''(x) = 2 + \cos x,\ f(0) = -1,\ f(\pi/2) = 0$

6. § The graph of f' is shown below. Sketch the graph of f assuming that f is continuous and $f(0) = -1$.



7. § A particle starts at $s(0) = 0$ and with velocity $v(0) = 5$, with an acceleration $a(t) = \cos t + \sin t$. Find the position of the particle as a function of time.

8. § A particle moves with an acceleration $a(t) = 10\sin t + 3\cos t$ in such a way that $s(0) = 0$ and $s(2\pi) = 12$. Find the position of the particle as a function of time.

9. § Prove that for motion in a straight line with constant acceleration a, initial velocity v_0 and initial displacement s_0, the displacement after time t is

$$s(t) = \frac{1}{2}at^2 + v_0 t + s_0$$

10. § A car is traveling at 100 km/h when the driver sees an accident 80 m ahead and slams on the brakes. What constant deceleration is required to stop the car in time to avoid a pileup?