Math 1A: Discussion Exercises

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Find two or three classmates and a few feet of chalkboard. As a group, try your hand at the following exercises. Be sure to discuss how to solve the exercises — *how* you get the solution is much more important than *whether* you get the solution. If as a group you agree that you all understand a certain type of exercise, move on to later problems. You are not expected to solve all the exercises: in particular, the last few exercises may be very hard.

Many of the exercises are from *Single Variable Calculus: Early Transcendentals for UC Berkeley* by James Stewart; these are marked with an §. Others are my own, or are independently marked.

The Fundamental Theorem of Calculus, part 1

1. § Find the *derivative* of each of the following functions (Hint: chain rule):

   (a) $g(x) = \int_1^x \frac{1}{t^3+1}\,dt$

   (b) $g(y) = \int_2^y t^2 \sin t\,dt$

   (c) $F(x) = \int_x^\pi \sqrt{1+\sec t}\,dt$

   (d) $G(x) = \int_x^1 \cos\sqrt{t}\,dt$

   (e) $h(x) = \int_2^{1/x} \arctan t\,dt$

   (f) $h(x) = \int_0^{x^2} \sqrt{1+r^3}\,dr$

   (g) $y = \int_0^{\tan x} \sqrt{t+\sqrt{t}}\,dt$

   (h) $y = \int_{1-3x}^1 \frac{u^3}{1+u^2}\,du$

   (i) $g(x) = \int_{2x}^{3x} \frac{u^2-1}{u^2+1}\,du$

   (j) $g(x) = \int_{\tan x}^{x^2} \frac{1}{\sqrt{2+t^4}}\,dt$

   (k) $y = \int_{\sqrt{x}}^{x^2} \sqrt{t}\,\sin t\,dt$

   (l) $y = \int_{\cos x}^{5x} \cos(u^2)\,du$

2. (a) Find the derivative of the following quantity:

   $$f(x) = \int_{\sin x}^{\cos x} \sqrt{1-v^2}\,dv$$

   (b) What is the general antiderivative of your answer to part (a)?

   (c) By interpreting the integral as an area, find $f(0)$.

   (d) Thus, find $f(x)$.

3. § Find a function f and a number a such that for every $x > 0$:

   $$6 + \int_a^x \frac{f(t)}{t^2}\,dt = 2\sqrt{x}$$

   First Hint: what is the derivative of the equation? Second Hint: when is the integral 0?