Math 1A: Discussion Exercises

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Find two or three classmates and a few feet of chalkboard. As a group, try your hand at the following exercises. Be sure to discuss how to solve the exercises — *how* you get the solution is much more important than *whether* you get the solution. If as a group you agree that you all understand a certain type of exercise, move on to later problems. You are not expected to solve all the exercises: in particular, the last few exercises may be very hard.

Many of the exercises are from *Single Variable Calculus: Early Transcendentals for UC Berkeley* by James Stewart; these are marked with an §. Others are my own, or are independently marked.

The Indefinite Integral

1. Let $f(x)$ be a continuous function. What is the derivative $\left(\int_a^b f(x)\, dx\right)'$? What is $\left(\int f(x)\, dx\right)'$? How does this illustrate the difference between definite and indefinite integrals?

2. What is wrong with the following:

$$\int_1^3 x\, dx = \frac{x^2}{2}\Big|_1^3 = \frac{9}{2} - \frac{1}{2} = \frac{8}{2} = \boxed{4 + C}$$

3. § Find the general indefinite integral:

   (a) $\displaystyle\int \left(\sqrt{x^3} + \sqrt[3]{x^2}\right) dx$

   (b) $\displaystyle\int \left(y^3 + 1.8y^2 - 2.4y\right) dy$

   (c) $\displaystyle\int v\left(v^2 + 2\right)^2 dv$

   (d) $\displaystyle\int \left(x^2 + 1 + \frac{1}{x^2 + 1}\right) dx$

   (e) $\displaystyle\int \left(\csc^2 t - 2e^t\right) dt$

   (f) $\displaystyle\int \sec t\,(\sec t + \tan t)\, dt$

   (g) $\displaystyle\int \frac{\sin 2x}{\sin x}\, dx$

   (h) $\displaystyle\int \left(\cos x + \frac{1}{2}x\right) dx$

   (i) $\displaystyle\int \left(e^x - 2x^2\right) dx$

4. § Evaluate the definite integral:

   (a) $\displaystyle\int_1^3 \left(1 + 2x - 4x^3\right) dx$

   (b) $\displaystyle\int_{-2}^0 \left(u^5 - u^3 + u^2\right) du$

   (c) $\displaystyle\int_0^9 \sqrt{2t}\, dt$

   (d) $\displaystyle\int_0^5 \left(2e^x + 4\cos x\right) dx$

   (e) $\displaystyle\int_{\pi/4}^{\pi/3} \sec\theta \tan\theta\, d\theta$

   (f) $\displaystyle\int_0^{\pi/3} \frac{\sin\theta + \sin\theta \tan^2\theta}{\sec^2\theta}\, d\theta$

   (g) $\displaystyle\int_{-10}^{10} \frac{2e^x}{\sinh x + \cosh x}\, dx$

   (h) $\displaystyle\int_0^{1/\sqrt{3}} \frac{t^2 - 1}{t^4 - 1}\, dt$

   (i) $\displaystyle\int_{-1}^2 \left(x - 2|x|\right) dx$

5. § Draw the curve $x = 2y - y^2$, and explain why the area between the curve and the y-axis is $\int_0^2 (2y - y^2)\, dy$. Evaluate this integral.

The Net Change Theorem

6. § If $w'(t)$ is the rate of growth of a certain child in pounds per year, what does $\int_5^{10} w'(t)\, dt$ represent?

7. § Recall that the marginal revenue function $R'(x)$ is defined as the derivative of the revenue function $R(x)$, which returns the total revenue received if a company sells x units of a given commodity. What does $\int_{1000}^{5000} R'(x)\,dx$ represent?

8. § If the units for x are feet and the units for $a(x)$ are pounds per foot, what are the units for da/dx? What about $\int_2^8 a(x)\,dx$?

9. § A particle starts with velocity $v(0) = -4$ and has acceleration $a(t) = 2t + 3$, where t is measured in seconds and $a(t)$ is in meters per second squared. For far does the particle travel in the first three seconds?

10. § The marginal cost of manufacturing x yards of a certain fabric is $C'(x) = 3 - 0.01\,x + 0.000\,006\,x^2$ (in dollars per yard). Find the increase in cost if the production level is raised from 2000 yards to 4000 yards.

11. § Find the interval $[a, b]$ for which the value of the interval $\int_a^b (2 + x - x^2)\,dx$ is a maximum.