Math 1A: Discussion Exercises

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Find two or three classmates and a few feet of chalkboard. As a group, try your hand at the following exercises. Be sure to discuss how to solve the exercises — *how* you get the solution is much more important than *whether* you get the solution. If as a group you agree that you all understand a certain type of exercise, move on to later problems. You are not expected to solve all the exercises: in particular, the last few exercises may be very hard.

Many of the exercises are from *Single Variable Calculus: Early Transcendentals for UC Berkeley* by James Stewart; these are marked with an §. Others are my own, or are independently marked.

Areas between curves

1. § Sketch the region enclosed by the given curves and find the area.

   (a) $y = \sin x$, $y = e^x$, $x = 0$, $x = \pi/2$

   (b) $y = x^2 - 2x$, $y = x + 4$

   (c) $y = x^2$, $y = 4x - x^2$

   (d) $y = 8 - x^2$, $y = x^2$, $x = -3$, $x = 3$

   (e) $y = x^2$, $y^2 = x$

   (f) $x = 2y^2$, $x = 4 + y^2$

   (g) $x = 1 - y^2$, $x = y^2 - 1$

   (h) $y = \sqrt{x}$, $y = \frac{1}{2}x$, $x = 9$

   (i) $4x + y^2 = 12$, $x = y$

   (j) $y = \sin(\pi x/2)$, $y = x$

   (k) $y = 3x^2$, $y = 8x^2$, $4x + y = 4$

   (l) $y = \cos x$, $y = \sin 2x$, $x = 0$, $x = \pi/2$

   (m) $y = x^2$, $y = 2/(x^2 + 1)$

   (n) $y = 1/x$, $y = x$, $y = \frac{1}{4}x$

2. § If the birth rate of a population is $b(t) = 2200e^{0.024t}$ people per year and the death rate is $d(t) = 1460e^{0.018t}$ people per year, where in each case t is the number of years since some given time, find the area between the curves $y = b(t)$ and $y = d(t)$ for $0 \leq t \leq 10$, and explain what the area represents.

3. Prove the following fact, first discovered (by other means) by Archimedes:

   Consider three equally spaced points on the curve $y = x^2$; let's say $P_1 = \big((a-b), (a-b)^2\big)$, $P_2 = \big(a, a^2\big)$, and $P_3 = \big((a+b), (a+b)^2\big)$. Then the area of the of the triangle connecting the three points P_1, P_2, P_3 has exactly three-quarters the area enclosed by the parabola and the line connecting P_1 and P_3.

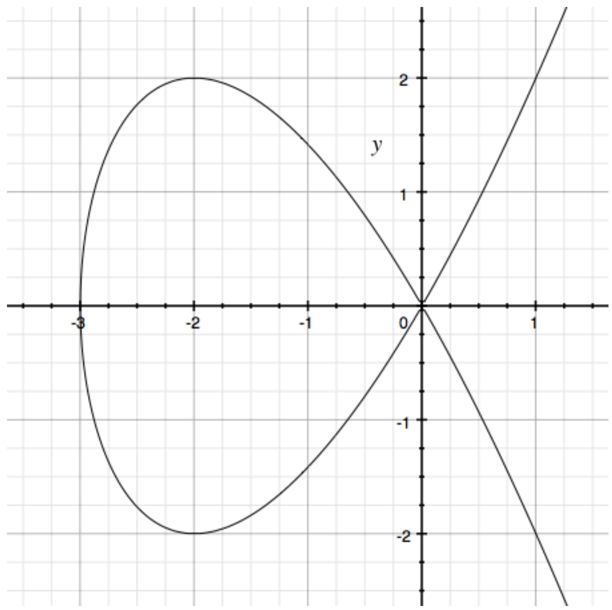
4. § Sketch the region in the x, y-plane defined by the inequalities $x - 2y^2 \geq 0$ and $1 - x - |y| \geq 0$; find its area.

5. § Find the area of the region bounded by the parabola $y = x^2$, the tangent line to this parabola at $(1, 1)$, and the x-axis.

6. § Consider the region R under the curve $y = 1/x^2$, $1 \leq x \leq 4$. Find a number a so that the line $x = a$ bisects the region R. Then find a number b so that the line $y = b$ bisects the region R.

7. § For what values of m do the line $y = mx$ and the curve $y = x/(x^2 + 1)$ enclose a region? Find the area of the region (your answer should be a function of m).

8. § Below is a graph of the curve $y^2 = x^2(x + 3)$. Find the area inclosed in the loop.



9. § Find a positive continuous function f such that the area under the graph of f from 0 to t is t^3 for all $t > 0$.

10. § For what m does the line $y = mx$ divide the region bounded by the parabola $y = x - x^2$ and the x-axis exactly in half?

11. § Suppose the graph of a cubic polynomial intersects the parabola $y = x^2$ when $x = 0$, $x = a$, and $x = b$, where $0 < a < b$. If the two regions between the curves have the same area, how is b related to a?