Math 1A: Discussion Exercises

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Find two or three classmates and a few feet of chalkboard. As a group, try your hand at the following exercises. Be sure to discuss how to solve the exercises — *how* you get the solution is much more important than *whether* you get the solution. If as a group you agree that you all understand a certain type of exercise, move on to later problems. You are not expected to solve all the exercises: in particular, the last few exercises may be very hard.

Many of the exercises are from *Single Variable Calculus: Early Transcendentals for UC Berkeley* by James Stewart; these are marked with an §. Others are my own, or are independently marked.

Volumes

1. § Sketch the following curves and the region enclosed. Find the volume of the solid formed by rotating the region around the given line.

   (a) $y = \frac{1}{4}x^2$, $y = 5 - x^2$; about the x-axis.

   (b) $x = y^2$, $x = 2y$; about the y-axis.

   (c) $y = x$, $y = \sqrt{x}$; about $y = 1$.

   (d) $y = 1 + \sec x$, $y = 3$; about $y = 1$.

   (e) $x = y^2$, $x = 1$; about $x = 1$.

   (f) $y = x^2$, $x = y^2$; about $x = -1$.

2. Prove the following theorem of Archimedes:

   A cylinder with the same height and diameter has the same total volume as the total volume of the sphere with the same diameter together with the cone with the same height and diameter.

3. § Find the volume of the right circular cone with height h and base r.

4. § Find the volume of the pyramid with height h and rectangular base with dimensions $b \times 2b$.

5. § Find the volume of the tetrahedron with three mutually perpendicular faces and three mutually perpendicular edges with lengths 3 cm, 4 cm, and 5 cm.

6. Use calculus to prove that $V = \frac{1}{3}Bh$, where V is the volume of a "pyradmid" whose base is some arbitrary shape of area B, whose height is h, and all of whose cross sections are similar and shrink linearly from the base to the vertex.

7. § Find the volume of the solid doughnut. This is the volume of revolution formed by rotating a circle of radius r centered at $(R, 0)$ around the y-axis, where $R > r > 0$.

8. § Find the volume of the solid whose base is the region enclosed by the parabola $y = 1 - x^2$ and the x-axis, and whose cross-sections perpendicular to the y-axis are squares.

9. (a) § Two cylinders with the same radius intersect at right angles. Find the volume of the intersection.

   (b) Three cylinders with the same radius intersect at right angles. Find the volume of the intersection.

10. § A bead is made by boring a cylindrical whole of radius r through a solid sphere of radius R (where $0 < r < R$). Find the volume of material of the bead.

11. § Two solids of revolution are formed by rotating the same region around different lines: namely, a region above the x-axis is rotated around the x-axis and around $y = -k$ for some constant $k > 0$. Prove that the difference between the volumes of the two solids depends only on k and on the area of the region, and find a formula for this dependence.

12. § A sphere of radius 1 intersects a smaller sphere of radius $r < 1$ in such a way that their intersection is a circle of radius r. In other words, they intersect in a great circle of the small sphere. Find r so that the volume inside the small sphere and outside the large sphere is as large as possible.