

You must always justify your answers. This means: show your work, show it neatly, and when in doubt, use words (and pictures!) to explain your reasoning. No justification = no points.

1. (4 pts) Evaluate the following limit, or explain why it doesn't exist:

$$\lim_{t \rightarrow 0} \left( \frac{1}{t} - \frac{1}{t^2 + 1} \right)$$

We know that any rational function is continuous in its domain, and so we can calculate limits by evaluating if the function is defined. Unfortunately, in the above function at  $t = 0$  we would need to divide by 0:

$$\frac{1}{0} - \frac{1}{0 + 1} = \frac{1}{0} - 1$$

This suggests that the limit does not exist. In fact, the function has a vertical asymptote at  $t = 0$ , as can be seen by graphing: note especially that  $1/(t^2 + 1)$  is everywhere defined.

On one side of the asymptote,  $1/t \rightarrow +\infty$ , and on the other side,  $1/t \rightarrow -\infty$ . Thus there is not even an infinite limit.

2. (6 pts) Is the function  $f(t) = \frac{1}{t} - \frac{1}{t^2 + 1}$  continuous at  $t = 0$ ? Is it continuous at  $t = 1$ ? At  $t = 2$ ? At each of these values of  $t$ , if  $f(t)$  is not continuous, does it have a removable discontinuity, a jump discontinuity, an infinite discontinuity, or an essential discontinuity?

As we said above, the function is continuous wherever it is defined, and in particular it is continuous at  $t = 1$  and  $t = 2$ . At  $t = 0$ , the function has a vertical asymptote, and hence an infinite discontinuity.

3. (bonus) On the back of this page, explain a concept from this course that you feel like you completely understand, but that you didn't understand a week ago.