Math 1A: Quiz 3

ANSWERS

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Wednesday, 11 February 2008

You must always justify your answers. This means: show your work, show it neatly, and when in doubt, use words (and pictures!) to explain your reasoning. No justification = no points.

1. (4 pts) For what value of c is the function $f(x)$ continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx^2 + 3x - 1 & \text{if } x < 1 \\ c(x-2) + 4 & \text{if } x \geq 1 \end{cases}$$

Each piece of the function is continuous, using the Continuity Rules for polynomials. I.e. $f(x)$ is continuous on $(-\infty, 1) \cup (1, \infty)$ no matter what c is. So the only question is the point $x = 1$. For f to be continuous at 1, we need the left-hand limit and the right-hand limit to agree.

$$\begin{aligned} \lim_{x \to 1^-} f(x) &= \lim_{x \to 1^-} \left(cx^2 + 3x - 1\right) \\ &= c \cdot 1^2 + 3 \cdot 1 - 1 = c + 2 \end{aligned}$$

$$\begin{aligned} \lim_{x \to 1^+} f(x) &= \lim_{x \to 1^+} (c(x-2) + 4) \\ &= c(1-2) + 4 = 4 - c \end{aligned}$$

So the possible cs that make f continuous correspond to solutions to $c + 2 = 4 - c$. I.e. $\boxed{c = 1}$ is the only answer.

2. (6 pts) Assume that you've found a constant c so that $f(x)$ from the previous problem is continuous on $(-\infty, \infty)$. Use the intermediate value theorem to show that the equation $f(x) = 0$ has a solution in the interval $[0, 2]$. Be sure to state the conditions of the theorem, and why $f(x)$ satisfies the conditions.

We know that $f(0) = c \cdot 0^2 + 3 \cdot 0 - 1 = -1$ and $f(2) = c(2-2) + 4 = 4$, so 0 is an intermediate value between $f(0)$ and $f(2)$. Moreover, $f(x)$ is continuous on $[0, 2]$ by assumption (the statement of the question essentially says that we should assume that we have solved question 1, and we have set c to the solution). Hence the conditions of the Intermediate Value theorem are satisfied, and thus there is a point $x \in [0, 2]$ such that $f(x) = 0$.

3. (bonus) On the back of this page, explain a concept from this course that you don't understand, but explain what you don't understand about it well enough that someone not in this course can understand the question.