Math 1A: Quiz 5 *ANSWERS*

GSI: Theo Johnson-Freyd Wednesday, 25 February 2008

You must always justify your answers. This means: show your work, show it neatly, and when in doubt, use words (and pictures!) to explain your reasoning. No justification = no points.

1. (6 pts) Find an equation of the tangent line at the point $(0, 1)$ to the following curve:

$$y = \frac{\cos x}{\cos x - \sin x}$$

We differentiate, using the quotient rule and the derivatives of sine and cosine.

$$\begin{aligned} y' &= \frac{\cos' x(\cos x - \sin x) - \cos x(\cos x - \sin x)'}{(\cos x - \sin x)^2} \\ &= \frac{\sin x(\cos x - \sin x) - \cos x(-\sin x - \cos x)}{(\cos x - \sin x)^2} \end{aligned}$$

At $x = 0$, this is

$$y'(0) = \frac{0(1-0) - 1(-0-1)}{(1-0)^2} = 1$$

Thus, we want a line with slope 1 passing through the point $(0, 1)$. The equation is:

$$\boxed{y = x + 1}$$

2. (4 pts) Find the derivative of the following function:

$$f(x) = 2^{\sqrt{x}}$$

We differentiate with the chain rule. The outer function is $f = 2^u$, with derivative $\ln 2\, 2^u$, and the inner function is $u = \sqrt{x}$, with derivative $u' = \frac{1}{2}x^{-1/2} = 1/2\sqrt{x}$. Thus,

$$f'(x) = \ln 2\, 2^x \frac{1}{2} x^{-1/2} = \boxed{\frac{\ln 2}{2} \frac{2^x}{\sqrt{x}}}$$

3. (bonus) On the back of this page, describe something you like about this discussion section, and something you don't like.