Math 1A: Quiz 12 Name: _____

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You must always justify your answers. This means: show your work, show it neatly, and when in doubt, use words (and pictures!) to explain your reasoning. No justification = no points.

1. (4 pts) Evaluate $\int_0^2 \left(2 + \sqrt{4 - x^2}\right) dx$ by interpreting the definite integral as an area. Do not use the Fundamental Theorem of Calculus.

2. (6 pts) Find an expression for $\int_1^3 e^x \, dx$ as a limit of sums. Do not evaluate the expression.

3. (bonus) Without using the Fundamental Theorem of Calculus, evaluate the limit from question 2. *First Hint:* You may use the following fact without proof:

$$\sum_{i=1}^{n} a^i = a + a^2 + \cdots + a^n = \frac{a^{n+1} - a}{a - 1} = a\frac{a^n - 1}{a - 1} \text{ for any } a \neq 1$$

Second Hint: You will probably need to use L'Hospital's Rule. When you do, you may find that it's easier to first make a substitution: $u = \frac{1}{n}$, $\lim_{n\to\infty} f(n) = \lim_{u\to 0^+} f(1/u)$.