Math 1A: Quiz 13

ANSWERS

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Wednesday, 29 April 2009

You must always justify your answers. This means: show your work, show it neatly, and when in doubt, use words (and pictures!) to explain your reasoning. No justification = no points.

1. (5 pts) Find the *derivative* of the following function:

$$g(x) = \int_0^{2x+1} \sqrt{\sin t}\, dt$$

We write $g(x) = f(h(x))$, where $h(x) = 2x+1$ and $f(u) = \int_0^u \sqrt{\sin t}\, dt$. By the fundamental theorem of calculus:

$$f'(u) = \sqrt{\sin u}$$

Thus, by the chain rule:

$$g'(x) = f'(h(x))\, h'(x) = \sqrt{\sin(2x+1)}\,(2x+1)' = \boxed{2\sqrt{\sin(2x+1)}}$$

2. (5 pts) Find the following definite integral:

$$\int_{-1}^{2} (x^2+1)\, dx$$

We use the power rule to find an antiderivative of x^2+1: the derivative of x^3 is $3x^2$, so the derivative of $\frac{x^3}{3}$ is x^2, and the derivative of x is 1. Therefore $\frac{x^3}{3}+x$ is an antiderivative of x^2+1; it is continuous and differentiable on $[-1,2]$. By the fundamental theorem of calculus, $\int_{-1}^{2}(x^2+1)\,dx = \left[\frac{x^3}{3}+x\right]_{-1}^{2} = \frac{2^3}{3}+2-\left(\frac{(-1)^3}{3}+(-1)\right) = \frac{8}{3}+2-\frac{-1}{3}-(-1) = \frac{9}{3}+3 = \boxed{6}$.

3. (bonus) On the back of this page, find $\displaystyle\int_{-1}^{1} \frac{1}{x^2+1}\, dx$.

We know that $\arctan'(x) = 1/(x^2+1)$, and so $\int_{-1}^{1} \frac{1}{x^2+1}\, dx = \arctan(1) - \arctan(-1) = \frac{\pi}{4} - (-\frac{\pi}{4}) = \boxed{\frac{\pi}{2}}$, since arctan is continuous on $[-1,1]$.