Math 1A: Quiz 14 ANSWERS

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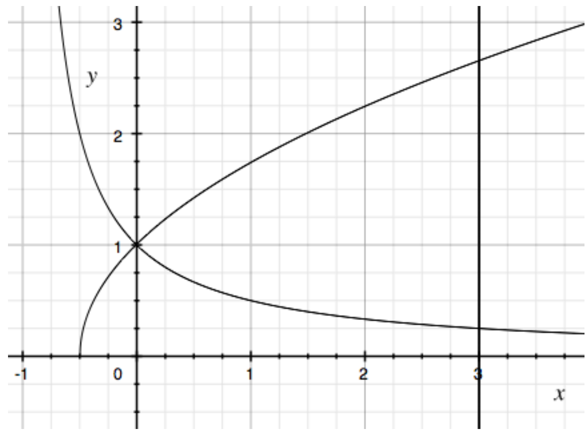
You must always justify your answers. This means: show your work, show it neatly, and when in doubt, use words (and pictures!) to explain your reasoning. No justification = no points.

1. (10 pts) Graph the curves $y = 1/(1+x)$, $y = \sqrt{2x+1}$, and $x = 3$. Find the enclosed area.

   The graph is below. It's easy enough to notice that the two curves $y = 1/(1+x)$ and $y = \sqrt{2x+1}$ intersect that $(0,1)$.



   Then the area of enclosed is

$$\int_0^3 \left(\sqrt{2x+1} - \frac{1}{1+x}\right) dx = \int_0^3 \sqrt{2x+1}\, dx - \int_0^3 \frac{1}{1+x}\, dx$$

   We compute the first integral by substituting $u = 2x+1$, whence $du = 2dx$ and so $dx = \frac{1}{2}du$, and so

$$\int_0^3 \sqrt{2x+1}\, dx = \int_1^7 \sqrt{u}\, \frac{du}{2} = \frac{1}{2}\frac{u^{3/2}}{3/2}\Bigg|_1^7 = \frac{1}{3}\left(7^{3/2} - 1^{3/2}\right) = \frac{7\sqrt{7}-1}{3}$$

   The second integral is also a straightforward substitution with $u = 1+x$, so $dx = du$: $\int \frac{dx}{1+x} = \int duu = \ln u + C = \ln(1+x) + C$, so the definite integral is $\ln(1+3) - \ln(1+0) = \ln 4$. All together, we have:

$$\text{Area} = \int_0^3 \sqrt{2x+1}\, dx - \int_0^3 \frac{1}{1+x}\, dx = \boxed{\frac{7\sqrt{7}-1}{3} - \ln 4}$$

2. (bonus) On the back of this page, sketch the curve $y^2 = x(x+1)(x+2)$ and write an expression for the area of the closed loop. You do not need to evaluate the expression.

   We have $y = \pm\sqrt{x(x+1)(x+2)}$. The function $y = x(x+1)(x+2)$ is positive for $-2 \leq x \leq -1$ and for $0 \leq x$, and so $\pm\sqrt{x(x+1)(x+2)}$ is defined only on these intervals. In fact, the graph of $y = \pm\sqrt{x(x+1)(x+2)}$ is essentially a circle passing through $(-2,0)$ and $(-1,0)$, and also a strange curve in the positive-x half-plane. The area of the closed loop is twice the integral of the top half: $\boxed{2\int_{-2}^{-1} \sqrt{x(x+1)(x+2)}\, dx}$.