Math 1B: Discussion Exercises

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Find two or three classmates and a few feet of chalkboard. As a group, try your hand at the following exercises. Be sure to discuss how to solve the exercises — *how* you get the solution is much more important than *whether* you get the solution. If as a group you agree that you all understand a certain type of exercise, move on to later problems. You are not expected to solve all the exercises: some are very hard.

Most of today's handout is originally from Rob Bayer.

Power Series Solutions to Differential Equations

1. Rewrite (i.e. re-index) each of the following expressions to get each into the form $\sum d_n x^n$:

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+2}}{2^{n+1}} x^{n+3}$ (b) $\displaystyle\sum_{n=1}^{100} n x^{n-1}$ (c) $\displaystyle\sum_{n=2}^{\infty} \frac{n(n-1)}{2} x^{n-2}$

2. Suppose $y(x) = \sum_{n=0}^{\infty} c_n x^n$. Express y', y'', xy, and xy' as power series of the form $\sum d_n x^n$. (I.e. in each answer, the nth term in the sum should be a number (depending on n and the c_ms) times x^n.)

3. Let $y = \sum_{n=0}^{\infty} c_n x^n$. In each of the following expressions, substitute this power series and re-write the results as a power series of the form $\sum d_n x^n$:

   (a) $y' - 6y$ (b) $xy'' - y$

4. Unwind the following recurrence relations:

   (a) $(n+1)a_{n+1} = a_n$. Write a_n in terms of a_0.

   (b) $c_{n+2} = -\dfrac{c_n}{(n+1)(n+2)}$. Write c_n in terms of c_0 or c_1.

   (c) $n^2 a_n + a_{n-2} = 0$ and $a_1 = 0$. Write a_n in terms of a_0.

5. Use power series to find the general solution to $y' - y = 0$. Is the answer what you expected?

6. Use power series to solve the differential equation:

   (a) $y' = xy$ (b) $(x-2)y' + 2y = 0$ (c) $y'' = y$

   Also solve each of the above equations without using power series. In this way, derive power series representations for all those functions. Find the interval of convergence of each of the above power series.