

Math 1B: First Exam
Tuesday, 24 July 2009

Instructor: Theo Johnson-Freyd
<http://math.berkeley.edu/~theo/f/09Summer1B/>

Name: _____

| Problem Number | 1 | 2 | 3 | 4 | 5 | Total |
|----------------|----|----|----|----|----|-------|
| Score | | | | | | |
| Maximum | 20 | 20 | 20 | 20 | 20 | 100 |

Please do not begin this test until 2:10 p.m. You may work on the exam until 4 p.m.
Please do not leave during the last 15 minutes of the exam time.

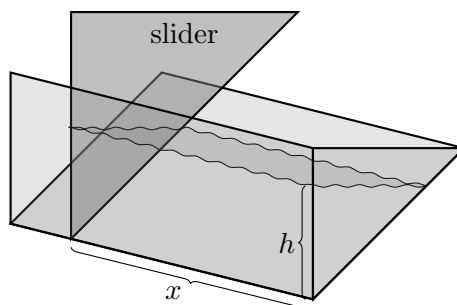
You must always justify your answers: show your work, show it neatly, and when in doubt, use words (and pictures!) to explain your reasoning. Please box your final answers.

Calculators are not allowed. Please sign the following honor code:

I, the student whose name and signature appear on this midterm, have completed the exam by myself, without any help during the exam from other people, or from sources other than my allowed one-page hand-written cheat sheet. Moreover, I have not provided any aid to other students in the class during the exam. I understand that cheating prevents me from learning and hurts other students by creating an atmosphere of distrust. I consider myself to be an honorable person, and I have not cheated on this exam in any way. I promise to take an active part in seeing to it that others also do not cheat.

Signature: _____

2. A trough is constructed with vertical ends and flat sides that meet at a 45° point, so that each cross section is a right triangle, as shown in the picture. One end of the trough can move by sliding along the trough.



- (a) (10 pts) If the trough is filled with water to a depth h , find the hydrostatic force on one end of the trough.

- (b) (5 pts) Enough water is poured into the trough so that when the sliding end is such that the trough is one meter long, the water comes to a depth of ten centimeters. If you move the slider so that the trough has length x , how deep will the water now be?

- (c) (5 pts) The *work* required to move the slider from a length a to a length b is defined to be $W = \int_b^a F(x)dx$, where $F(x)$ is the hydrostatic force on the slider when the slider is at length x . Find the work required to shrink the trough from a length of one meter to a length of fifty centimeters.

3. The class sizes at a certain large university are distributed exponentially: in a given semester there are $N(x) = Ae^{-x/B}$ classes of size x , where A and B are positive constants. Since universities are large, we approximate x and $N(x)$ (which actually can only take integer values) by continuous variables.

(a) (5 pts) How many classes does the university offer in a given semester?

(b) (5 pts) Write a probability distribution expressing the probability that a given class has size x .

(c) (5 pts) What is the average class size at this university?

(d) (5 pts) Jimmy Stewart is a student at this university. Since the probability that he ends up in any particular class is proportional to the number of students in that class, the probability that his first-period class has size x is proportional to $xAe^{-x/B}$. Find the expected size of Jimmy's first-period class.

4. A certain population with constant carrying capacity K has a relative growth rate $k(t)$ that varies sinusoidally with time: $k(t) = a \sin(bt + c) + d$ for constants a, b, c, d .

(a) (3 pts) Assuming that the relative growth rate is always positive, what can you say about the coefficients a, b, c, d ?

(b) (2 pts) Write a differential equation modeling the population growth.

- (c) (15 pts) Assuming that the initial ($t = 0$) population is P_0 , find a formula for the population at time t .

5. (20 pts) A spring has mass $m = 1$ kg, damping constant $c = 2$ kg/s, and spring constant $k = 2$ kg/s², and the spring is driven by a force $F(t) = \sin(t/s)$ kg m/s², where t is the time. Find the general solution describing the displacement of spring as a function of time.

References: All the problems on this midterm are due to the instructor, although they are loosely based on the material in *Single Variable Calculus: Early Transcendentals for UC Berkeley* by James Stewart. The honor-code language is adapted from the Stanford Honor Code (<http://www.stanford.edu/dept/vpsa/judicialaffairs/guiding/honorcode.htm>) and from the exams by Zvezda Stankova.

Feel free to use this page for extra scrap work.