Math 1B: Quiz 1

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ANSWERS

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You must always justify your answers. This means: show your work, show it neatly, and when in doubt, use words (and pictures!) to explain your reasoning. No justification = no points.

1. (5 pts) Evaluate the integral $\int_0^4 y\, e^{-y/2}\, dy$.

   We integrate by parts, taking $u = y$ and $dv = e^{-y/2}dy$, whence $v = \frac{e^{-y/2}}{-1/2} = -2e^{y/2}$:

$$\begin{aligned}\int_0^4 y\, e^{-y/2}\, dy &= \left[y(-2)e^{-y/2}\right]_0^4 - \int_0^4 (-2)e^{-y/2}\, dy \\ &= 4(-2)e^{-4/2} - 0(-2)e^0 + 2\int_0^4 e^{-y/2}\, dy \\ &= -8e^{-2} + 2\left[(-2)e^{-y/2}\right]_0^4 \\ &= -8e^{-2} - 4e^{-2} + 4e^0 \\ &= \boxed{4 - 12e^{-2}}\end{aligned}$$

2. (5 pts) Evaluate the integral $\int \tan^3\theta \sec^5\theta\, d\theta$.

   We recognize $\tan\theta\sec\theta\, d\theta = d(\sec\theta)$, and $\tan^2\theta = \sec^2\theta - 1$. Thus, we substitute $u = \sec\theta$:

$$\begin{aligned}\int \tan^3\theta \sec^5\theta\, d\theta &= \int \left(\sec^2\theta - 1\right)\sec^4\theta\ \ \tan\theta\sec\theta\, d\theta \\ &= \int \left(u^2 - 1\right)u^4\, du \\ &= \int \left(u^6 - u^4\right) du \\ &= \frac{u^7}{7} - \frac{u^5}{5} + C \\ &= \boxed{\frac{\sec^7\theta}{7} - \frac{\sec^5\theta}{5} + C}\end{aligned}$$

3. (0 pts) Can you make my currently scheduled office hours? Would you rather have Monday and/or Wednesday office hours from 4 to 4:45? Consider that moving Monday's office hours later means no office hours before the quiz on Monday.