

Math 1B: Optional Quiz

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ANSWERS

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Give yourself 15 minutes for this quiz.

1. Determine whether the integral $\int_2^3 \frac{dx}{\sqrt{3x-x^2}}$ converges or diverges. Prove your answer.

The denominator is zero at $x = 0$ and $x = 3$; only $x = 3$ is in the domain. The integrand is $1/\sqrt{x(3-x)}$, and when $x \approx 3$, this is $1/\sqrt{3(x-3)}$. The integral $\int_2^3 1/\sqrt{x-3} dx = \int_2^3 (x-3)^{-1/2} dx$ converges by the p-test. So we expect that the original integral also converges. Thus, let us try to bound it above so that we can use the comparison test.

When $x \in [2, 3]$, we know that $1/\sqrt{x} \leq 1$. So:

$$\frac{1}{\sqrt{3x-x^2}} = \frac{1/\sqrt{x}}{\sqrt{3-x}} \leq \frac{1}{\sqrt{3-x}}$$

Moreover, $1/\sqrt{x(3-x)}$ is positive for $x \in [2, 3)$. Thus we can use the comparison test. The integral $\int_2^3 (3-x)^{-1/2} dx = \int_0^1 u^{-1/2} du$ with $u = 3-x$, which definitely converges by the p-test, as $1/2 < 1$. \square