Math 1B: Quiz 4 ANSWERS

GSI: Theo Johnson-Freyd Tuesday, 14 July 2009

You must always justify your answers. This means: show your work, show it neatly, and when in doubt, use words (and pictures!) to explain your reasoning. No justification = no points.

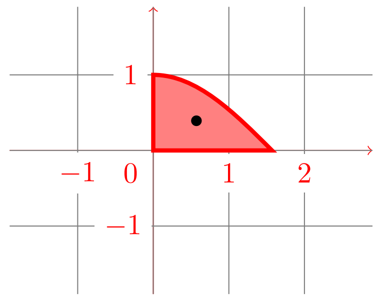
1. (5 pts) Sketch the region $R = \{y \leq \cos x;\ y \geq 0;\ x \leq \pi/2\}$. Find the coordinates of the centroid of the region R, and mark it on your sketch.

We use the formulas for the centroid of a region:

$$A = \int_0^{\pi/2} \cos x\, dx = \boxed{1}$$

$$\begin{aligned}\bar{x} &= \frac{1}{A}\int_0^{\pi/2} x\cos x\, dx \\ &= [x\sin x]_0^{\pi/2} - \int_0^{\pi/2}\sin x\, dx \\ &= \boxed{\frac{\pi}{2} - 1}\end{aligned}$$

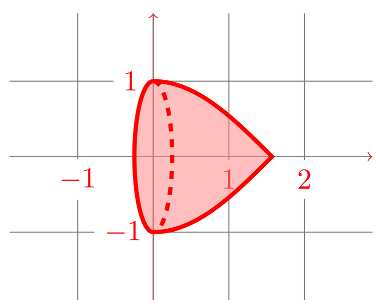
$$\begin{aligned}\bar{y} &= \frac{1}{A}\int_0^{\pi/2} \frac{1}{2}\cos^2 x\, dx \\ &= \frac{1}{1}\frac{1}{2}\frac{\pi}{2}\frac{1}{2} = \boxed{\frac{\pi}{8}}\end{aligned}$$



2. (5 pts) Sketch the solid of revolution formed by rotating the region $R = \{y \leq \cos x;\ y \geq 0;\ x \leq \pi/2\}$ around the x-axis. Find the *total* surface area of the solid.

The total surface area consists of a circular piece with area π and a lateral piece with area:

$$\begin{aligned}\mathrm{SA} &= \int_0^{\pi/2} 2\pi\cos x\sqrt{1+\sin^2 x}\, dx \\ &= 2\pi\int_0^1 \sqrt{1+u^2}\, du \\ &= 2\pi\left[\frac{u}{2}\sqrt{1+u^2} + \frac{1}{2}\ln\left(u+\sqrt{1+u^2}\right)\right]_0^1 \\ &= 2\pi\left(\frac{1}{2}\sqrt{2} + \frac{1}{2}\ln\left(1+\sqrt{2}\right)\right)\end{aligned}$$



We have used the surface-area formula with $f(x) = \cos x$ (whence $f'(x) = -\sin x$), and substituted $u = \sin x$. We conclude that the total area is:

$$\pi + 2\pi\left(\frac{1}{2}\sqrt{2} + \frac{1}{2}\ln\left(1+\sqrt{2}\right)\right) = \boxed{\pi\left(1+\sqrt{2}+\ln(1+\sqrt{2})\right)}$$