Math 1B: Quiz 8 ANSWERS

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You must always justify your answers. This means: show your work, show it neatly, and when in doubt, use words (and pictures!) to explain your reasoning. No justification = no points.

1. (10 pts) Determine whether the following series converges or diverges. If it converges, find the limit.

$$\sum_{n=1}^{\infty}\left(\cos\frac{\pi}{n+1}-\cos\frac{\pi}{n}\right)$$

We compute the Nth partial sum s_N:

$$\begin{aligned} s_N = \sum_{n=1}^{N}\left(\cos\frac{\pi}{n+1}-\cos\frac{\pi}{n}\right) &= \left(\cos\frac{\pi}{2}-\cos\frac{\pi}{1}\right)+\left(\cos\frac{\pi}{3}-\cos\frac{\pi}{2}\right)+\cdots+ \\ &\quad +\left(\cos\frac{\pi}{N-1+1}-\cos\frac{\pi}{N-1}\right)+\left(\cos\frac{\pi}{N+1}-\cos\frac{\pi}{N}\right) \\ &= -\cos\frac{\pi}{1}+\cos\frac{\pi}{N+1} \end{aligned}$$

Thus, the infinite series converges if and only if $\lim_{N\to\infty} s_N$ exists, and this limit gives the value. We have:

$$\begin{aligned} \lim_{N\to\infty} s_N &= \lim_{N\to\infty}\left(-\cos\pi+\cos\frac{\pi}{N+1}\right) \\ &= -\cos\pi+\lim_{N\to\infty}\cos\frac{\pi}{N+1} \\ &= -(-1)+\cos 0 = \boxed{2} \end{aligned}$$

In particular, the series $\boxed{\text{converges}}$.

2. (bonus) Explain why it's wrong to write:

$$\sum_{n=1}^{\infty}\left(\cos\frac{\pi}{n+1}-\cos\frac{\pi}{n}\right)=\left(\sum_{n=1}^{\infty}\cos\frac{\pi}{n+1}\right)-\left(\sum_{n=1}^{\infty}\cos\frac{\pi}{n}\right)$$

We cannot write the above equation because the sums on the right-hand-side each diverge. Indeed,

$$\lim_{n\to\infty}\cos\frac{\pi}{n+1}=\cos 0=1$$

and so the sums each fail the divergence test.