Math 1B: Quiz 10

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ANSWERS

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You must always justify your answers. This means: show your work, show it neatly, and when in doubt, use words (and pictures!) to explain your reasoning. No justification = no points.

1. (5 pts) Determine whether the following series converges absolutely, converges conditionally, or diverges. You do not need to find the limit. You must specify which convergence test(s) you are using, and why the conditions for the test are satisfied.

$$\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2+1}$$

The series is alternating, so we write $\frac{(-1)^n n}{n^2+1} = a_n = (-1)^n b_n$ with $b_n = n/(n^2+1)$. Then $\lim b_n = 0$ by L'H, and b_n is decreasing, as

$$\left(\frac{x}{x^2+1}\right)' = \frac{1-x^2}{(x^2+1)^2} \leq 0$$

Thus the conditions of the AST are satisfied, and the series converges.

On the other hand, the absolute series $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ diverges by LCT with $1/n$. Thus the original series if $\boxed{\text{conditionally convergent}}$.

2. (5 pts) Determine whether the following series converges absolutely, converges conditionally, or diverges. You do not need to find the limit. You must specify which convergence test(s) you are using, and why the conditions for the test are satisfied.

$$\sum_{n=1}^{\infty} \frac{n(-3)^n}{n!}$$

We use the ratio test.

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(n+1)(-3)^{n+1}/(n+1)!}{n(-3)^n/n!}\right| = \left|\frac{(-3)^{n+1}}{(-3)^n}\frac{n+1}{n}\frac{n!}{(n+1)!}\right| = 3\frac{n+1}{n}\frac{1}{n+1} = \frac{3}{n} \to 0$$

Since the limit is $0 < 1$, the series $\boxed{\text{converges absolutely}}$.