Math 1B: Quiz 12

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ANSWERS

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You must always justify your answers. This means: show your work, show it neatly, and when in doubt, use words (and pictures!) to explain your reasoning. No justification = no points.

1. (10 pts) Find a power series representation (centered at 0) of solution to the following initial value problem:

$$xy'' + y' + y = 0, \;\; y(0) = 1, \;\; y'(0) = -1$$

We begin by assuming that $y(x) = \sum_0^\infty c_n x^n$ for some coefficients c_n. Then $y' = \sum_0^\infty (n+1)c_{n+1}x^n$ and $xy'' = \sum_0^\infty n(n+1)c_{n+1}x^n$ in the usual way. Thus, the power series for the LHS is:

$$\sum_{n=0}^{\infty} \left(n(n+1)c_{n+1} + (n+1)c_{n+1} + c_n \right) x^n$$

and this must be a power series representation for 0. Thus, the coefficients must each vanish:

$$(n+1)^2 c_{n+1} + c_n = 0 \text{ for every } n$$

We are given that $y(0) = 1$ and $y'(0) = -1$, and so $c_0 = 1$ and $c_1 = -1$. Then

$$c_2 = -\frac{c_1}{2^2} = \frac{1}{2^2}$$
$$c_3 = -\frac{c_2}{3^2} = -\frac{1}{2^2 3^3}$$
$$c_4 = -\frac{c_3}{4^4} = \frac{1}{2^2 3^2 4^2}$$

We can guess the pattern:

$$c_n = \frac{(-1)^n}{(n!)^2}$$

Thus, our final answer is

$$\boxed{y(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n!)^2} x^n}$$