Math 2135: Linear Algebra

Final exam - practice

Your name:

University academic honour statement:

Dalhousie University has adopted the following statement, based on "The Fundamental Values of Academic Integrity" developed by the International Center for Academic Integrity (ICAI):

> Academic integrity is a commitment to the values of learning in an academic environment. These values include honesty, trust, fairness, responsibility, and respect. All members of the Dalhousie community must acknowledge that academic integrity is fundamental to the value and credibility of academic work and inquiry. We must seek to uphold academic integrity through our actions and behaviours in all our learning environments, our research, and our service.

Please **sign this page** to confirm that you will uphold these values, and that the work you submit on this exam is your own.

Exam structure

Part A contains six short unrelated questions, worth five points each.
Part B contains one longer question worth ten points.

Part A.

This section presents you with six statements. **Every one of them is false.** Give a short (one or two sentence) explanation of why.

1. Suppose $U \subset V$ is a vector subspace, and $w_1, \ldots, w_m$ is a basis for the quotient space V/U. Choose elements $v_1, \ldots, v_m \in V$ such that $w_1 = v_1 + U$, $\ldots$, $w_m = v_m + U$. Then $v_1, \ldots, v_m$ spans V.

2. If V is a vector space and $T \in \mathcal{L}(V)$ is injective, then T is invertible.

3. Suppose V is a finite-dimensional vector space and $T \in \mathcal{L}(V)$ is a linear operator. Then the set of eigenvectors of T is a vector subspace of V.

4. Suppose that V is a vector space, $T \in \mathcal{L}(V)$ is an operator, and $v_1, \ldots, v_m \in V$ are (nonzero) eigenvectors of T with eigenvalues $\lambda_1, \ldots, \lambda_n$. If $v_1, \ldots, v_m$ are linearly independent, then $\lambda_1, \ldots, \lambda_m$ are distinct.

5. Suppose that V is a finite-dimensional vector space, $T \in \mathcal{L}(V)$, and $U_1 \subset V$ is a T-invariant vector subspace. Then there exists a T-invariant vector subspace $U_2 \subset V$ such that $V = U_1 \oplus U_2$ is a direct sum decomposition.

6. There exists an operator $T \in \mathcal{L}(\mathbb{R}^3)$ such that the only T-invariant subspaces of $\mathbb{R}^3$ are the trivial ones $\{0\}$ and $\mathbb{R}^3$.

Part B.

Find all values of x such that the matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & x \end{pmatrix}$$

is diagonalizable. Justify your answer.