Math 2135: Linear Algebra

Midterm exam

1 March 2022

Your name:

University academic honour statement:

Dalhousie University has adopted the following statement, based on "The Fundamental Values of Academic Integrity" developed by the International Center for Academic Integrity (ICAI):

> Academic integrity is a commitment to the values of learning in an academic environment. These values include honesty, trust, fairness, responsibility, and respect. All members of the Dalhousie community must acknowledge that academic integrity is fundamental to the value and credibility of academic work and inquiry. We must seek to uphold academic integrity through our actions and behaviours in all our learning environments, our research, and our service.

Please **sign this page** to confirm that you will uphold these values, and that the work you submit on this exam is your own.

Exam structure

Part A contains six short unrelated questions, worth five points each.
Part B contains one longer question worth ten points.

Part A.

The next three pages presents you with six statements. Every one of them is false. For each one, give a short (one or two sentences) explanation of why.

1. The empty set is a vector space.

2. If V is a vector space and $\{v_1, \ldots, v_m\}$ is a linearly independent set of vectors in V, and $\{w_1, \ldots, w_n\}$ is a spanning set of V, then $m < n$.

3. There exists a linear transformation $\mathbb{R}^7 \to \mathbb{R}^4$ with two-dimensional kernel.

4. Suppose $T : V \to W$ is a linear transformation, and $\{v_1, \ldots, v_n\}$ is a basis for V. If $\{Tv_1, \ldots, Tv_n\}$ spans W, then T is injective.

5. Suppose V_1 and V_2 are finite-dimensional vector space. Then $\dim(V_1 \times V_2) = \dim(V_1) \times \dim(V_2)$.

6. If $T : V \to W$ is a linear transformation, and $U \subseteq V$ is a vector subspace, then there exists a linear transformation $\bar{T} : V/U \to W$ defined by $\bar{T}(v + U) = T(v)$.

Part B.

Let $V = \mathbb{R}^{\leq 2}[x]$ be the vector space of polynomials of degree ≤ 2. (This vector space is denoted $\mathcal{P}_2(\mathbb{R})$ in the textbook.) For each real number $r \in \mathbb{R}$, let $\mathrm{ev}_r : V \to \mathbb{R}$ denote the linear functional that sends $f(x) \mapsto f(r)$.

Show that the set $\{x^2 - 1, x^2 - x, x^2 + x\}$ is a basis for V, and that the set $\{-\mathrm{ev}_0, \frac{1}{2}\mathrm{ev}_{-1}, \frac{1}{2}\mathrm{ev}_1\}$ is its dual basis.

Remark: Partial marks will be provided for a discussion of what needs to be proved, even if you don't know how to prove some steps.