Math 2135: Linear Algebra

Practice Midterm

Your name:

University academic honour statement:

Dalhousie University has adopted the following statement, based on "The Fundamental Values of Academic Integrity" developed by the International Center for Academic Integrity (ICAI):

> Academic integrity is a commitment to the values of learning in an academic environment. These values include honesty, trust, fairness, responsibility, and respect. All members of the Dalhousie community must acknowledge that academic integrity is fundamental to the value and credibility of academic work and inquiry. We must seek to uphold academic integrity through our actions and behaviours in all our learning environments, our research, and our service.

Please **sign this page** to confirm that you will uphold these values, and that the work you submit on this exam is your own.

Exam structure

Part A contains six short unrelated questions, worth five points each.
Part B contains one longer question worth ten points.

Part A.

The next three pages presents you with six statements. Every one of them is false. For each one, give a short (one or two sentences) explanation of why.

1. The number 0 is an element of every vector space.

2. $V = \mathbb{C}$ can be thought of as a vector space over both $\mathbb{F} = \mathbb{R}$ and $\mathbb{C}$, and it has the same dimension no matter which $\mathbb{F}$ you choose.

3. If V is a vector space and $U_1, U_2 \subset V$ are vector spaces, then the union $U_1 \cup U_2$ is also a vector subspace of V.

4. Suppose $T : V \to W$ is a linear transformation, and $\ker T = \{0\}$. Then for every $w \in W$, there is exactly one vector $v \in V$ such that $Tv = w$.

5. If V is a finite-dimensional vector space with dual vector space V^*, then for any element $\phi \in V^*$,

$$\dim(V) = \dim(\ker \phi) + 1.$$

6. If V is a vector space, then $\{0\} \subset V$ is a vector subspace. However, the quotient $V/\{0\}$ is invalid because you cannot divide by 0.

Part B.

For any real number x, let $\beta(x) := \frac{x(x-1)}{2}$. In other words, $\beta(x) = \binom{x}{2}$ is "the number of ways to choose 2 things from x things," even if x is not a natural number.

Consider the set $\mathbb{R}^2$. Let's give it a "funny vector addition," called $+_f$, defined by

$$(x_1, y_1) +_f (x_2, y_2) := (x_1 + x_2 + y_1 y_2, y_1 + y_2)$$

Let's also give it a "funny scalar multiplication," called $\cdot_f$, defined by

$$\lambda \cdot_f (x, y) := (\lambda x + \beta(\lambda) y^2, \lambda y)$$

It turns out that these funny operations do make $\mathbb{R}^2$ into a vector space. Prove that the function $T(x, y) := (x + \beta(y), y)$ is an isomorphism between $\mathbb{R}^2$ with the usual operations and $\mathbb{R}^2$ with the funny operations.