Math 3032: Abstract Algebra

Assignment 5

due March 23, 2023

1. Find a Gröbner basis for the ideal $\langle x^2y - x - 2, xy + 2y - 9\rangle \subset \mathbb{R}[x, y]$ with respect to the ordering $x \gg y$. Describe, in as much detail as you can, the corresponding algebraic variety, i.e. the set of solutions to the system of equations $x^2y - x - 2 = xy + 2y - 9 = 0$.

2. Find a Gröbner basis for the ideal $\langle x^2y + x + 1, xy^2 + y - 1\rangle \subset \mathbb{R}[x, y]$ with respect to the ordering $x \gg y$. Describe, in as much detail as you can, the corresponding algebraic variety, i.e. the set of solutions to the system of equations $x^2y + x + 1 = xy^2 + y - 1 = 0$.