PhD Comprehensive Exam: Algebraic Topology (nonspecialist) & Math 4180/5180 Final Exam

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Practice Sample Exam

Your name:

Exam structure:

There are 6 questions on this exam. The pass mark is 70%.

- The PhD comprehensive exam consists of all 6 questions. You have three hours to complete the comprehensive exam.
- The Math 4180/5180 final exam consists of any 4 of the 6 questions. You have two hours to complete the final exam.

Please indicate which exam you are taking.

1. (a) State the definition of *Euler characteristic* of a finite CW complex.

   (b) Prove that there is no convex polyhedron all of whose faces are 6-gons.

2. Prove that a covering space induces an injection on π_1 and an isomorphism on π_n for $n > 1$.

3. Prove that any two continuous maps $S^2 \to S^1 \times S^1$ are homotopic.

4. Is the map $f : \{x \in \mathbb{R} \text{ s.t. } x > 0\} \to \{z \in \mathbb{C} \text{ s.t. } \|z\|^2 = 1\}$ defined by $f(x) = e^{2\pi i x}$ a covering map? Why or why not?

5. Let A be a simplex in $\mathbb{R}^n$ for some n, and let $B \subset A$ be a union of some of the faces of A. Prove that the homology groups $\mathrm{H}_m(B)$ are isomorphic to the homology groups $\mathrm{H}_{m+1}(A, B)$ for all $m > 0$. Give a counterexample to the corresponding statement if $m = 0$ and A is a 1-simplex.

6. (a) Compute the fundamental group of the suspension of $\mathbb{R}P^2$. Justify your answer.
   (b) Compute the homology groups of the suspension of $\mathbb{R}P^2$. Justify your answer.
   (c) Compute the $\mathbb{F}_2$-cohomology ring of the suspension of $\mathbb{R}P^2$. Justify your answer.
   (d) Compute the second homotopy group of the suspension of $\mathbb{R}P^2$. Justify your answer.