

# Math 3032: Abstract Algebra

Midterm exam

7 March 2023

**Your name:**

## **University academic honour statement:**

Dalhousie University has adopted the following statement, based on “The Fundamental Values of Academic Integrity” developed by the International Center for Academic Integrity (ICAI):

Academic integrity is a commitment to the values of learning in an academic environment. These values include honesty, trust, fairness, responsibility, and respect. All members of the Dalhousie community must acknowledge that academic integrity is fundamental to the value and credibility of academic work and inquiry. We must seek to uphold academic integrity through our actions and behaviours in all our learning environments, our research, and our service.

Please **sign here** to confirm that you will uphold these values, and that the work you submit on this exam will be your own.

## **Exam structure**

Part A contains four short unrelated questions, worth five points each.

Part B contains one longer question worth ten points.

**Part A.**

1. State the distributive law in ring theory.

2. Give an example of a unique factorization domain which is not a principal ideal domain.

**Note:** A correct example without justification will receive 4/5 points. For the last point, explain why your example is a UFD and not a PID. An incorrect example will receive zero points.

3. Consider the following statement:

If  $R$  is a commutative ring and  $J \subset R$  is an ideal, then there exists a commutative ring homomorphism  $\varphi : R \rightarrow S$  with  $\ker(\varphi) = J$ .

Either show that this statement is true by giving an example of such a homomorphism  $\varphi$ , or show that this statement is false by giving an example of a ring  $R$  with an ideal  $J$  for which no such homomorphism exists.

4. Is there a field  $F$  and an injective ring homomorphism  $\mathbb{Z}_6 \hookrightarrow F$ ? If so, describe such an  $F$ , and if not, explain why not.

**Part B.**

Prove that  $x^3 + 2x^2 + 3 \in \mathbb{Q}[x]$  is irreducible.