Math 3032: Abstract Algebra

Practice Midterm

7 March 2023

Your name:

University academic honour statement:

Dalhousie University has adopted the following statement, based on "The Fundamental Values of Academic Integrity" developed by the International Center for Academic Integrity (ICAI):

> Academic integrity is a commitment to the values of learning in an academic environment. These values include honesty, trust, fairness, responsibility, and respect. All members of the Dalhousie community must acknowledge that academic integrity is fundamental to the value and credibility of academic work and inquiry. We must seek to uphold academic integrity through our actions and behaviours in all our learning environments, our research, and our service.

Please **sign this page** to confirm that you will uphold these values, and that the work you submit on this exam is your own.

Exam structure

Part A contains four short unrelated questions, worth five points each.
Part B contains one longer question worth ten points.

Part A.

1. Suppose that $f(x), g(x) \in \mathbb{Z}_3[x]$, and that, by performing long division, you write $f(x) = q(x)g(x) + r(x)$. The ideal $(f(x), g(x))$ is necessarily equal to one of the following:

$$(f(x), r(x)), \qquad (g(x), r(x)), \qquad (f(x), q(x)), \qquad (g(x), q(x)), \qquad (q(x), r(x))$$

   Which one?

2. In the following sentence, should the blank be filled in with the word "all," the words "some but not all," or the words "none of the"?

   In the ring $\mathbb{C}[x, y]$, ____________ maximal ideals are principal.

   You do not need to justify your answer.

3. If $\varphi : R \to S$ is a ring homomorphism, is it necessarily true that $S \cong R/\ker(\varphi)$? If no, what extra condition on φ is needed to make it true?

4. Suppose that F is a field and $\varphi : R \to F$ is a ring homomorphism. Which of the following statements are necessarily true? You do not need to justify your answers.

- If φ is injective, then R is commutative.
- If φ is injective, then R is an integral domain.
- If φ is injective, then the ideal $\ker(\varphi)$ is maximal.
- If φ is surjective, then the ideal $\ker(\varphi)$ is maximal.
- If φ is surjective, then F is isomorphic to the field of fractions of R.

Part B.

Let p be a prime. Prove that $x^4 - 4x^3 + 6x^2 + 11x + 6 \in \mathbb{Q}[x]$ is irreducible. Hint: substitute $x \mapsto x + 1$.