1 Problem Set 1: "Preventive Medicine" Due February 20, 2008

****For posterity's sake, I will also type up the handed-out problem sets.****

Let $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} s & 0 \\ 0 & t \end{pmatrix}$, for $s, t \geq 0$.

1. Determine for which s, t we have $B \geq A$.
2. Determine for which s, t we have $B \geq A^+$. ****Recall from lecture: A^+ is the positive part of A, where we consider A as a function on its spectrum.****
3. Find values of s, t for which $B \geq A$, $B \geq 0$, and yet $B \not\geq A^+$. (So be careful about false proofs.)
4. Find values of s, t such that $B \geq A^+ \geq 0$ and yet $B^2 \not\geq (A^+)^2$. (So again be careful.)
5. Can you find values of s, t such that $B \geq A^+$ and yet $B^{1/2} \not\geq (A^+)^{1/2}$?
6. For 2×2 matrices T and P such that $T \geq 0$ and P is an orthogonal projection, is it always true that $PTP \leq T$?