Problem Set # 1

PI Lectures on Finite Symmetry in Field Theory

June 13, 2022

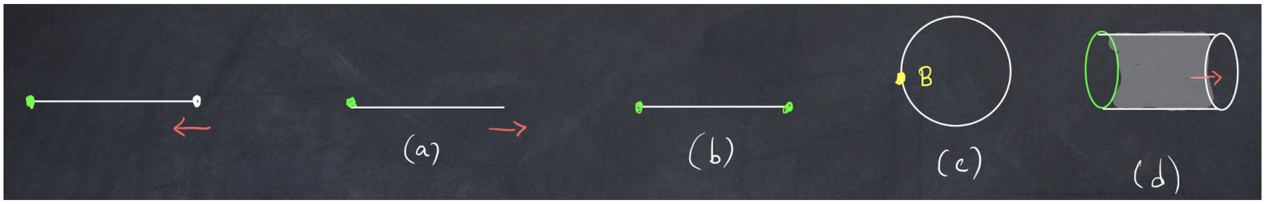

FIGURE 1. Some bordisms in the topological field theory (σ, ρ)

1. Let G be a finite group, and let $\sigma\colon \mathrm{Bord}_2 \to \mathcal{C}$ be the 2-dimensional finite gauge theory. You can take $\mathcal{C} = \mathrm{Alg}_1(\mathrm{Vect})$ the Morita 2-category of complex algebras, or $\mathcal{C} = \mathrm{Cat}$ a 2-category of complex linear categories. In the former case $\sigma(\mathrm{pt}) = \mathbb{C}[G]$ is the group algebra of G; in the latter case $\sigma(\mathrm{pt}) = \mathrm{Rep}(G)$ is the category of linear representations of G. Let ρ be the right regular boundary theory; then the first bordism in Figure 1 evaluates to the right regular module A_A (or to the functor $\mathrm{Rep}(G) \to \mathrm{Vect}$ which maps a G-module to its underlying vector space.) Let B be an (A, A)-bimodule. The red arrow indicates incoming vs. outgoing boundary components. Compute the value of (σ, ρ) on the bordisms (a), (b), (c), and (d) in Figure 1. You may need to specify more data to achieve an unambiguous answer.

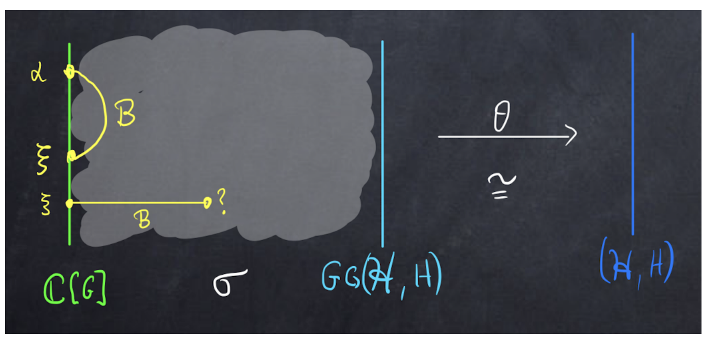

FIGURE 2. Two defects in quantum mechanics

2. Figure 2 uses the pictorial notation from the lecture; see also the lecture notes. Here we are working with the 2-dimensional finite G-gauge theory of Problem 1, which acts on a quantum mechanical system given by a Hilbert space $\mathcal{H}$ and a Hamiltonian H. In the figure, B is a (dualizable) (A, A)-bimodule, $\xi \in B$ is a vector, and $\alpha \in B^*$ is a functional. Take the vertical line to be imaginary time. The bottom defect is at a fixed time, but data is missing at the right endpoint. What data goes there? (It is an element of $\mathrm{Hom}\big(1, \sigma(L)\big)$, where L is the link of the point.) What is the image of that defect under θ? (Note that it is a (σ, ρ)-defect.) For the top defect, suppose that ξ, α are at some times t_1, t_2. Compute the image of this defect under θ.