Problem Set # 3

PI Lectures on Finite Symmetry in Field Theory

June 15, 2022

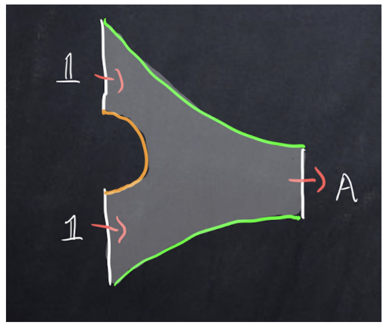

FIGURE 1. An element in $\mathrm{Hom}(1, A)$

1. Let G be a finite group, let $\chi\colon G \to \mathbb{C}^\times$ be a character (it could be the trivial character that sends each $g \in G$ to $1 \in \mathbb{C}^\times$), and let $\sigma = \sigma^{(2)}_{BG}$ be the 2-dimensional finite G-gauge theory with values in $\mathcal{C} = \mathrm{Alg}(\mathrm{Vect})$ the Morita 2-category of algebras. Then $\sigma(\mathrm{pt}) = A = \mathbb{C}[G]$. The regular boundary theory (constructed from the regular module A_A) is indicated in Figure 1 in green, and the augmentation boundary theory in orange. The depicted bordism maps to an element of $\mathrm{Hom}(1, A)$, where this is Hom in $\Omega\mathcal{C} = \mathrm{Hom}_{\mathcal{C}}(1, 1) = \mathrm{Vect}$. (Why?) So it can be identified with an element of $A = \mathbb{C}[G]$. Compute that element.

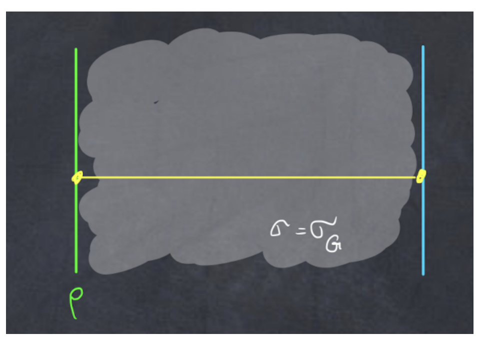

FIGURE 2. Point defects in quantum mechanics

2. Let $(\mathcal{H}, H)$ define a quantum mechanical theory F which is invariant under the action of a finite group G. Let σ be the 2-dimensional G-gauge theory and consider Figure 2, which shows F in the "sandwich" picture. There a general point defect is depicted. Point defects in F form a vector space. What extra structure is encoded in this sandwich picture? Now replace the right regular boundary ρ with a quotient $\epsilon_{G'}$ for a subgroup $G' \subset G$: it is associated to the right module $\mathbb{C}\langle G' \backslash G \rangle$ for the group algebra $A = \mathbb{C}[G]$. What theory is obtained? What are the point defects in terms of the point defects for F? What happens if you twist by a character of G'?