THE MU PUZZLE

The following exercises are based on Chapter 1 of:

- Douglas R. Hofstadter, *Gödel, Escher, Bach: An Eternal Golden Braid*, Basic Books, 1979.

Basic vocabulary and grammar of Miu

The purpose of this exercise is to study an artificial language called "Miu." It has three words:

Definition (word): A *word* in Miu is any of the following three symbols (when writing in Miu, we will always use a non-serif font):

$$M, \quad I, \quad U$$

You should think of these as logographs (e.g. Japanese kanji or Chinese characters) rather than symbols from an alphabet (e.g. Cyrillic) or syllabary (e.g. Japanese kana). An English-to-Miu dictionary would have only three entries. I won't provide an English-to-Miu dictionary (indeed, I don't own one), but some of the meanings of Miu will become apparent.

Definition (string): A *string* (also called a *formula*) is any list of Miu words.

Examples: The following are all strings:

$$M, \quad IUI, \quad UU, \quad MMMUM$$

On the other hand, XMA is not a string in Miu, since neither X nor A is a word in Miu.

We can now define the *grammar* of our artificial language.

Definition (grammatical sentence): A string is a *grammatical sentence* (also called a *well-formed formula*) if its first word is M, and no other words are Ms.

Examples: The following strings are grammatical sentences:

$$M, \quad MIUIU, \quad MUUUIII$$

The following strings are not grammatical:

$$U, \quad UIUMM, \quad MUM$$

Thus the *meaning* of the word M is purely *syntactic*: it means "this is the start of a sentence (and the end of any previous sentence)." **Note:** *Syntax* is essentially the same as *grammar*. For comparison, *semantics* means "meaning" — the things "in the real world" that words signify. The words I and U are purely *semantic*.

We will call any list of grammatical sentences a *paragraph*.

Exercise 1: Which strings are paragraphs? Is there unambiguous *parsing* (i.e. breaking of a paragraph into sentences)?

Sound arguments

In any language, not all grammatical sentences express truths. To convince someone of the truth of a sentence, you should give a *sound argument.* Recall that an argument is *valid* if every step follows logically from the previous steps. But that's not good enough: you could start with a false premise. An argument is *sound* if it is both valid and also every premise is true.

In Miu, there is one sentence that everyone knows to be true (agreed-upon sentences are called *axioms*):

Axiom: MI

Since we don't know what I means, we don't know what this sentence means — it could mean, for example, "There is no God but Allah," or maybe it means "0 = 0."

There are four *rules of inference*, which are the allowed steps in a valid argument.

Rule 1: If x is any string, then it is valid to proceed from MxI to MxIU.

Note: The italic letter "x" is not a string in Miu. What the rule means is that if you choose any string in Miu, and replace all instances of x with that string, then you get an instance of the rule.

Example: It is valid to go from the claim MUUI to the conclusion MUUIU.

Rule 2: If x is any string, then from Mx it is valid to conclude Mxx.

Example: MUMUU is a valid paragraph. I.e. you may validly follow the sentence MU by the sentence MUU.

Rule 3: If III occurs anywhere in a sentence, it is valid to replace it by U.

Example: MUIIIIUMUIUU and MUIIIIUMUUIU are both valid paragraphs.

Rule 4: If UU occurs anywhere in a sentence, it is valid to delete it.

Example: MUUUUMUUM is a valid paragraph consisting of three sentences.

All together, a paragraph is *sound* if it is valid and also its first sentence is MI.

Exercise 2: Come up with some sound arguments to get a feel for the rules of inference.

Justifiable knowledge

Definition (justifiable): A sentence is *justifiable* if there is some sound argument (the *justification*) with that sentence as its conclusion. (Justifiable sentences are also called *theorems*; their justifications are also called *proofs*.)

Example: MIIUIIU is justifiable, because the paragraph MIMIIMIIUMIIUIIU is sound.

Main exercise: Is MU justifiable?