SVOAs and some exceptional groups

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Main classification result:

If V is an $N=1$ SVOA s.t.:

(1) V_{ev} is a s.c. WZW model

(2) $V_{1/2} = 0$.

(3) $V \not\supseteq E_7$ or E_8

Then V is one of:

[Noether: $V_{1/2} = \mathrm{Lie}\,\mathrm{Aut}_{N=1} V$.]

[suffice: $V \not\supseteq E_{7,2}$ as a subalg]

V_{ev}	$\dim V_{3/2}$	c	$\mathrm{Aut}_{N=1}(V)$
$Spin(m)_3$	$\frac{m(m-1)(m+4)}{6}$	$\frac{3m(m-1)}{2(m+1)}$	S_{m+1}
$Spin(m)_1^3$	m^3	$\frac{3m}{2}$	$2^{2(m-1)}:(S_3 \times S_m)$ ↳ enhances to $3S_6$ when $m=4$.
$Sp(3)_2$	84	7	$U_3(3):2$
$Sp(3)_1^2$	196	$8\frac{2}{5}$	$J_2:2$
$SU(6)_2$	175	$8\frac{3}{4}$	$M_{21}:2^2$
$Sp(6)_1$	429	$9\,3/4$	$G_2(4):2$
$SU(6)_1^2$	400	10	$U_4(3):D_8$
$Spin(12)_2$	462	11	$M_{12}:2$
$SU(12)_1$	924	11	$Suz:2$
$Spin(12)_1^2$	1024	12	$2^{10}:M_{12}:2$
$Spin(16)_1 \times Spin(8)_1$	1024	12	$2^8 \cdot O_8^+(2).2$
$Spin(24)_1$	2048	12	Co_1

I'll say a bit about the details of the statement in a moment. But what I really want to tell you is why I find it. See, this is the type of really fun project where you get to sit and calculate for awhile. Does it matter?

Probably not. But it was fun.

Why do I care? An ongoing question in exceptional group theory is: where do exceptional groups come from? The answer is often somewhat disappointing. Some groups, e.g. J_3, don't seem to come from anywhere: Janko just found it by identifying a large subgp. More typically, a sporadic gp is the automorphisms of some clever, but complicated, combinatorial object, e.g. HS = Aut(some highly-symmetric graph on 100 vertices). One sporadic gp, though, is particularly <u>fundamental</u>: Conway's largest gp Co_1.

Conway found Co_1 as $\mathrm{Aut}(\text{Leech lattice})/\pm 1$. $\subseteq PSO(24)$
But he already showed it is more fundamental: it is
$$\mathrm{Aut}(II_{25,1}) / \mathrm{Weyl}(II_{25,1}) = \mathrm{Out}(II_{25,1}).$$

I got interested by yet another appearance of Co_1, which is so far essentially unrelated to these lattice ones:

<u>Thm (Duncan):</u>
$Co_1 = \mathrm{Aut}_{N=1}($ unique $N=1$ SVOA w/
- $c=12$
- no free fermions
- holomorphic, i.e. $\mathrm{Rep}(V) = (s)\mathrm{Vec}$)

$=$ "$V^{f\natural}$"

For this $V^{f\natural}$, $V_{ev} = Spin(24)_1$.

Let me explain some of these words. Given this conference, I won't try to define VOA (and in any case I really want "nice" ones, e.g. unitary CFT-type...). An <u>$N=1$ structure</u> is a choice of <u>superconformal vector</u> $\tau \in V$ which should be a Virasoro primary of spin $3/2$ s.t.
$$\tau(z)\tau(0) \sim \frac{\frac{2}{3}c}{z^3} + \frac{\nu(0)}{z}$$
ν = conformal vector.