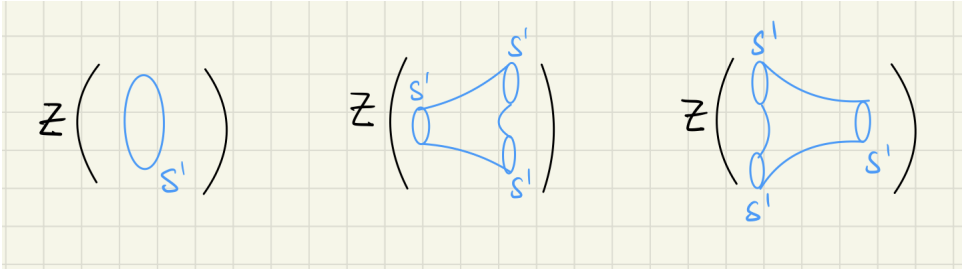
1 Problem 1

Consider a 1+1d $\mathbb{Z}_N$ gauge theory. Compute the following maps



2 Problem 2

Consider a 2+1d untwisted $\mathbb{Z}_N$ gauge theory. It can be described by the action

$$S = \frac{iN}{2\pi} a \wedge db \,, \tag{2.1}$$

where a and b are $U(1)$ one-form gauge fields. Compute the correlation function of line operators

$$\left\langle \exp\left(i \oint_{\gamma_1} a\right) \exp\left(i \oint_{\gamma_2} b\right) \right\rangle \,. \tag{2.2}$$

(Hint: it is related to the linking number of γ_1 and γ_2)

3 Problem 3

Consider an $SU(2)$ gauge theory in 3+1d. What is its one-form symmetry? How does it act on the Wilson lines $\mathrm{Tr}_{\mathcal{R}}\left(P \exp(i \oint_\gamma A)\right)$ in representation $\mathcal{R}$?

4 Problem 4

If a $\mathbb{Z}_N$ symmetry is spontaneously broken, there are N degenerate vacua. What is the analogous statement for $\mathbb{Z}_N$ one-form symmetry?

To answer this question, let us consider $U(1)$ gauge theory with a charge N scalar field.
a) What is the electric one-form symmetry
b) Find a phase where the theory is gapped and the electric one-form symmetry is spontaneously broken. What is the IR theory? (Hint: it is a topological field theory)

5 Problem 5

Construct a instanton configuration for a $U(1)$ bundle on $S^2 \times S^2$. (Hint: first construct a monopole configuration for a $U(1)$ bundle with nontrivial first Chern-class $c_1 = \frac{1}{2\pi} F$ on S^2)