FACTORIZATION ALGEBRAS: DAY 2 EXERCISES

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1. Exercises

Question 1.1. Show that a locally constant factorization algebra on $\mathbb{R}^n$ determines a locally constant factorization algebra on $\mathbb{R}^m$ for any $m < n$.

Question 1.2. Write out and try to visualize the multiplication

$$(\mathbb{E}_n \circ \mathbb{E}_n)(k) \to \mathbb{E}_n(k)$$

for some small values of n and k.

Question 1.3. Show that $\mathbb{E}_1$-algebras are associative algebras.

Question 1.4. Show that $\mathbb{E}_\infty$-algebras are commutative algebras.

Question 1.5. Show that an $\mathbb{E}_\infty$-algebra in Cat is a symmetric monoidal category.

Question 1.6. Show that an $\mathbb{E}_1$-algebra in Cat is a monoidal category.

Question 1.7. Show that an $\mathbb{E}_2$-algebra in Cat is a braided monoidal category.

Question 1.8. Let X be a pointed topological space. Show that $\Omega^n X$ is an $\mathbb{E}_n$-algebra in spaces.

In fact, every (connected, grouplike) $\mathbb{E}_n$-algebra in spaces looks like $\Omega^n X$ for some X. This is called May's Recognition Principle.

1.1. Enveloping Algebras.

For the following exercise, recall the Chevalley-Eilenberg complex of a Lie algebra

$$C_\bullet(\mathfrak{h}) = (\mathrm{Sym}(\mathfrak{h}[1]), d)$$

where d is determined by the bracket on $\mathfrak{h}$.

Let $\mathfrak{g}$ be a Lie algebra over $\mathbb{R}$. Let $\mathfrak{g}^{\mathbb{R}}$ be the cosheaf valued in chain complexes on $\mathbb{R}$ assigning

$$\Omega^*_c(U) \otimes \mathfrak{g}$$

with differential d_{dR} to an open interval $U \subset \mathbb{R}$.

Question 1.9. Show that the assignment

$$U \mapsto H^\bullet(C_\bullet(\mathfrak{g}^{\mathbb{R}}(U)))$$

defines a factorization algebra on $\mathbb{R}$. Call it $\mathcal{U}(\mathfrak{g})$.

Question 1.10. Show that $\mathcal{U}(\mathfrak{g})$ is locally constant.

Question 1.11. Show that $\mathcal{U}(\mathfrak{g})$ is determined by the associative algebra $U\mathfrak{g}$, the universal enveloping algebra.