FACTORIZATION ALGEBRAS: DAY 4 EXERCISES

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1. Exercises

Question 1.1. Let V be a finite dimensional vector space. Let Z_V be the 1-dimensional TQFT determined by V. What is the corresponding associative algebra of observables, in terms of V?

Question 1.2. Let Z be an n-dimensional field theory. Show that $Z(S^{n-1})$ acts on $Z(N)$ for every n-manifold N.

Question 1.3. For G-gauge theory on M, we defined a Wilson loop operator. Given a representation of G, define a version of a Wilson loop operator on $\mathsf{Gauge}_G M$.

Remark 1.4. For G-gauge theory on M, the category of line operators should be

$$\mathsf{Rep}(G) \times \mathsf{Rep}(G^L)$$

where G^L is the Langlands dual group. $(SO(2n)^L = SO(2n)$, $SL(n)^L = PGL(n))$.

Question 1.5. Let A be an $\mathbb{E}_n$-algebra. For example, $A = \mathsf{Obs}^q$ the observables of a TQFT on $\mathbb{R}^n$. The line operators of this field theory should be a $\mathbb{E}_{n-1}$-monoidal category. As guesses, build two different categories out of A. Do these categories have any monoidal structure?

1.1. Classification of 3d TQFTs.

We saw that, up to reversing orientation and taking disjoint unions,

$\mathsf{Cob}(1)$ had a unique object P, the point, and
$\mathsf{Cob}(1)$ had a unique object S^1.

Question 1.6. Describe the set of objects of $\mathsf{Cob}(3)$.

Examples of objects in $\mathsf{Cob}(3)$ include the torus T^2 and the sphere S^2.

Question 1.7. Is there a way to build all other objects of $\mathsf{Cob}(3)$ from T^2 and S^2?

Note that objects of $\mathsf{Cob}(3)$ determine morphisms in $\mathsf{Cob}(2)$; we can view a closed 2-manifold as a cobordism from the emptyset to itself.

Question 1.8. As morphisms in $\mathsf{Cob}(2)$, what does the operation you thought of in Question 1.7 correspond to categorically? What does it tell you about a TQFT

$$Z\colon \mathsf{Cob}(2) \to \mathsf{Vect}(k)?$$

This breaking complicated 2-dimensional manifolds down into easy pieces (like S^2 and T^2) is very helpful. If we want a classification of 3-dimensional TQFTs, we would like to be able to do this in $\mathsf{Cob}(3)$.

Question 1.9. Can you think of a way to encode the operation from Question 1.7 into a categorical setting involving 3-dimensional bordisms? (Any guesses or ideas are fine, this one is to just get you thinking!)