SPT AND MANIFOLD INVARIANTS: EXERCISE 2

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1. Show that the way we define finite gauge theory gives a symmetric monoidal functor, i.e. gives a T(Q)FT.
2. Here is the definition of Euler theory: Fix a nonzero complex number λ. define that $F(Y) = \mathbb{C}$ for any closed $(n-1)$manifold. For a **closed** bordism X (bordism from $\emptyset$ to $\emptyset$, $F(X)$ is $\lambda^{\chi(X)}$ where $\chi(X)$ is the Euler number of X. For a geneal bordism X from Y_0 to Y_1, $F(X) = \lambda^{\chi(X)-\chi(Y_0)}$. Show that it defines a T(Q)FT. (the Euler characteristic satisfies a gluing formula)
3. Prove the category of duality data for an object y is either empty or all hom sets has unique element in the category.
4. A vector space is dualizable if and only if it is finite dimension.
5. $F : \mathcal{B} \to \mathcal{C}$ symmetric monoidal functor. If y is dualizable, then $F(y)$ is dualizable.
6. $\eta : F \to G$ natural transformation of symmetric monoidal functors. If y is dualizable, then $\eta(y)$ is an isomorphism.