Modular operads: Exercises Sheet 2

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Examples

1. Define an operad encoding commutative monoids. Does it admit a cyclic/modular structure? Does it satisfy a universal property?
2. (a) ($\star$) Show that the (framed) E_2 operad is aspherical (i.e. all homotopy groups except π_0 and π_1 of the space of operations vanish). Hint: Relate $E_2(n)$ to ordered configuration spaces. Use the fibration between configuration spaces which forgets a point.

   (b) ($\star\star$) Does the E_2-operad admit a cyclic structure? Hint: What would this imply homology? (We will see in the lecture that the framed E_2-operad has a cyclic structure).
3. Describe the action of $\mathbb{Z}_4$ by cyclic permutations on the set of operations in $\mathsf{As}(T_3)$ in terms of the generating binary operation. Can you say something about general T_n?
4. ($\star$) In case you know what an A_∞-algebra is. Generalize the definition of As to an cyclic operad with values in chain complexes encoding those.
5. Construct Vect-valued operads encoding Lie-algebras and Poisson-algebras. Are they cyclic?

Algebras

1. Let $\mathcal{F}\colon \mathcal{C} \to \mathcal{C}'$ be a symmetric monoidal functor between symmetric monoidal categories and $\mathcal{O}$ a set valued modular operad. Show that there is a canonical functor $\mathsf{Alg}_{\mathcal{O}}(C) \to \mathsf{Alg}_{\mathcal{O}}(C')$
2. Show that the category of algebras over a modular operad is a groupoid. Is this also true for cyclic and ordinary operads?
3. For a set valued cyclic operad $\mathcal{O}$ give an explicit description of cyclic algebras in terms of a non-degenerate cyclic pairing on algebras for the corresponding ordinary operad? This involves defining the term cyclic pairings.
4. Which objects of the (higher) categories Set, Cat, Top, Vect, sVect admit symmetric non-degenerate pairings. ($\star\star$) Answer the same question for the 2-category of $\mathbb{C}$-linear categories.
5. What are cyclic associative and commutative algebras in Vect?