Modular operads: Exercises Sheet 3

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Category valued operads

1. Is the canonical map $\mathsf{As} \to \mathsf{RBr}$ cyclic?

2. Show that π_0 of a category valued operad is a set valued operad. Describe π_0 of Br and RBr.

3. Argue that Br algebras are braided monoidal categories. What are algebras over RBr?

4. Proof point (1) and (2) in the Theorem describing the cyclic action on RBr.

5. The goal of this exercise is to construct examples of cyclic associative algebras in categories:

   (a) Argue that monoidal categories are algebras over the associative operad.

   (b) Let $\mathcal{C}$ be a rigid monoidal semi-simple linear category with finitely many isomorphism classes of simple objects. Show that the hom-pairing $\mathcal{C}(X^\vee, Y)$ is non-degenerate.

   (c) Show that a pivotal structure on $\mathcal{C}$ makes this pairing symmetric.

   (d) Argue that semi-simple pivotal categories are examples of cyclic associative algebras.

6. Similar to the previous exercise, argue that semi-simple ribbon categories are cyclic fE_2 algebras.

7. Give a description of Π_1 of the topological surface operad.

8. A 3-dimensional handlebody is a compact oriented 3-manifold with boundary constructed by gluing 3-balls along boundary disks.

   (a) Define a groupoid valued modular operad Hbdy whose groupoid of arity n-operations has objects connected handlebodies with $n+1$-disks embedded into their boundary and morphisms isotopy classes of compatible diffeomorphisms.

   (b) Show that there is a map of modular operads $\mathsf{Hbdy} \to \Pi_1(\mathsf{Surf})$ which is an equivalence when restricted to surfaces and handlebodies of genus zero.

   (c) Is it an equivalence in general?