What is going on with tensor models?

Renormalization and the melonic fixed point

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Introduction

Tensor models are particularly interesting because their large N expansion is dominated by melonic diagrams. First, it is richer than the large N limit of vector models (dominated by bubbles diagrams). Second, it is simpler than the planar limit of matrix models.

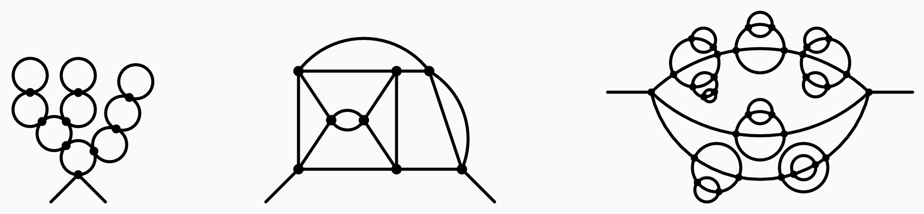

Fig. 1: Large N limit of vector, matrix and tensor models from left to right.

Tensor models were first studied in zero dimension in the context of random geometry and quantum gravity. They were then linked to quantum systems in one dimension. In particular, they provide an alternative to the SYK model without disorder. Recently, they were generalized to d dimensions and studied as proper quantum field theories. In this context, they give rise to a new family of conformal field theories (CFT). We call those type of CFT *melonic*. Thanks to the large N limit they can be studied both analytically and at strong coupling.

Model

The simplest model that can be studied is the quartic $O(N)^3$ bosonic tensor model. The bosonic field is a real tensor of rank 3 transforming under $O(N)^3$. Its indices are distinguished by the position. Contrary to scalar models, there are three quartic interactions (corresponding to the $O(N)^3$ invariants): tetrahedron, pillow and double trace. We represent them graphically:

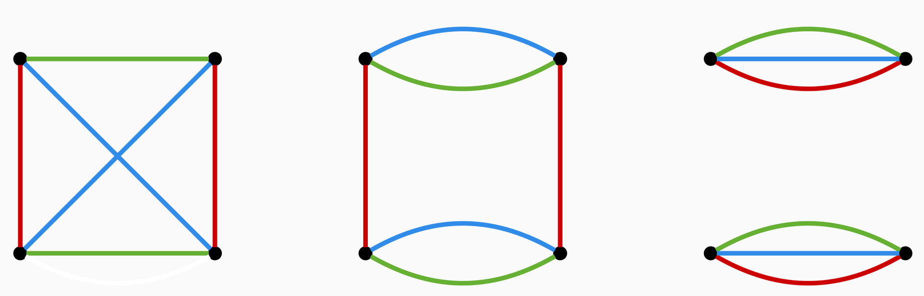

Fig. 2: We represent every tensor as a vertex and every contraction of two indices as an edge. The color of the edge corresponds to the position of the index.

We work at fixed dimension $d < 4$ and include a non trivial power of the Laplacian $0 < \zeta < 1$. It renders the interaction exactly marginal for $\zeta = \frac{d}{4}$. This kind of model is called *long-range*. To renormalize this model, we will consider the weakly relevant case $\zeta = \frac{d+\epsilon}{4}$ with small $\epsilon > 0$.

Flow and fixed points

We found four lines of fixed point parametrized by the tetrahedron coupling g. For a purely imaginary g, one of them is real and infrared stable [1].

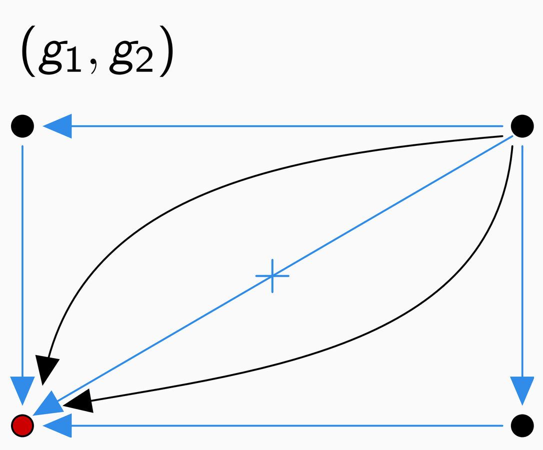

Fig. 3: Flow of the theory in the space of the pillow and double-trace couplings (g_1, g_2). The red dot is the infrared attractive fixed point.

Dimensions of bilinears and OPE Coefficients

In a CFT, the operator product expansion (OPE) is strongly constrained by conformal invariance and the sum restricts to primary operators:

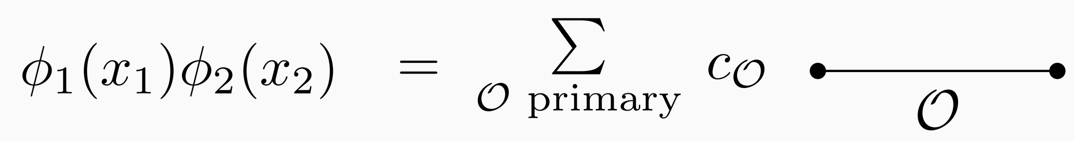

Fig. 4: Graphical representation of the OPE for a CFT.

where $c_{\mathcal{O}}$ are the OPE coefficients that we will now compute.
The four point function in a CFT can always be written as:

$$\langle\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \langle\phi(x_1)\phi(x_2)\rangle\,\langle\phi(x_3)\phi(x_4)\rangle + \\ + \sum_J \int_{\frac{d}{2}-\imath\infty}^{\frac{d}{2}+\imath\infty} \frac{dh}{2\pi\imath}\,\frac{1}{1-k(h,J)}\,\mu^d_{\Delta_\phi}(h,J)G^{\Delta_\phi}_{h,J}(x_i)$$

with J the spin, $G^{\Delta_\phi}_{h,J}(x_i)$ the conformal block, $\mu^d_{\Delta_\phi}(h,J)$ the measure and $k(h,J)$ the eigenvalues of the two particle irreducible four point kernel. The kernel is here known thanks to the large N limit.

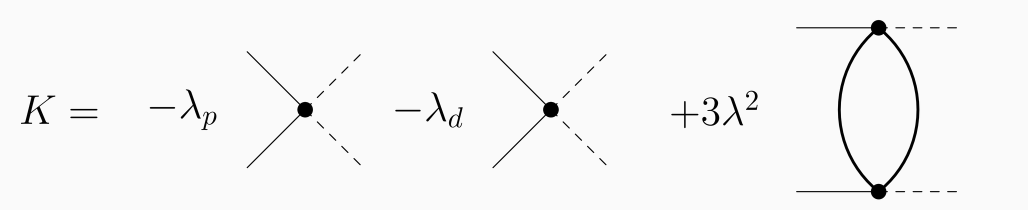

Fig. 5: Graphical representation of the kernel amputated to the right at leading order in N. The first two terms are based respectively on pillow and double-trace vertices while the last one is based on a pair of tetrahedral vertices.

The OPE coefficients are obtained by deforming the integration contour to the right and picking up the poles in the integrand. The only poles contributing are the $h_{m,J}$ such that $k(h,J) = 1$. These are exactly the dimensions of the bilinear operators $\phi\partial^J(\partial^2)^m\phi$. The squares of the OPE coefficients $c_{m,J}$ are then the residues at those poles.

Results

Dimensions of bilinears [2]:

- Spin $J = 0$: real for purely imaginary renormalized tetrahedral coupling g

$$h_\pm = \frac{d}{2} \pm 2\frac{\Gamma(d/4)^2}{\Gamma(d/2)}\sqrt{-3g^2} + \mathcal{O}(g^3)$$

- Spin $J \geq 0$, $m \in \mathbb{N}$: always real

$$h_{m,J} = \frac{d}{2} + J + 2m \\ - \frac{\Gamma(d/4)^4\Gamma(m+J)\Gamma(m+1-\frac{d}{2})\sin\left(\frac{\pi d}{2}\right)}{\Gamma(\frac{d}{2}+J+m)\Gamma(m+1)\,\pi}6g^2 + \mathcal{O}(g^4)$$

OPE coefficients:

- $(J,m) \neq (0,0)$: real OPE coefficients

$$c^2_{m,J} = 2\frac{\Gamma(J+\frac{d}{2})\Gamma(J+m)\Gamma(\frac{d}{2}+J+2m-1)}{\Gamma(J+1)\Gamma(m+1)\Gamma(1+2m-\frac{d}{2})\Gamma(\frac{d}{2}+J+m)} \\ \times \frac{\Gamma(1+m-\frac{d}{4})\Gamma(\frac{d}{4}+J+m)^2}{\Gamma(J+2m)\Gamma(\frac{d}{2}+2J+2m-1)\Gamma(\frac{d}{4}-m)^2}$$

- $(J,m) = (0,0)$: real for purely imaginary g

$$c^2_{0,0} = 2 \pm \sqrt{-3g^2}\frac{4\,\Gamma(d/4)^2}{\Gamma(\frac{d}{2})}\Big[2\Psi(d/4) - \Psi(d/2) - \Psi(1)\Big]$$

Those results point towards a unitary CFT at the IR stable fixed point in the large N limit.

Further work and Applications

- Proof of conformal invariance and correlation functions for composite operators [3].
- Computation of $1/N$ corrections: breaking of unitarity [4].
- Similar studies for other tensor models: sextic interactions, fermionic fields [5].
- Further study of long-range models: useful in statistical physics [6].

References

[1] D. Benedetti, R. Gurau, and S. Harribey. "Line of fixed points in a bosonic tensor model". In: *JHEP* (2019).
[2] D. Benedetti, R. Gurau, S. Harribey, and K. Suzuki. "Hints of unitarity at large N in the $O(N)^3$ tensor field theory". In: *JHEP* (2020).
[3] D. Benedetti, R. Gurau, and K. Suzuki. "Conformal symmetry and composite operators in the $O(N)^3$ tensor field theory". In: *JHEP* (2020).
[4] D. Benedetti, R. Gurau, and S. Harribey. "The tri-fundamental quartic model". In: (2020). arXiv: 2011.11276.
[5] D. Benedetti, N. Delporte, S. Harribey, and R. Sinha. "Sextic tensor field theories in rank 3 and 5". In: *JHEP* (2020).
[6] D. Benedetti, R. Gurau, S. Harribey, and K. Suzuki. "Long-range multi-scalar models at three loops". In: *J. Phys. A* (2020).