Higher Dualizability in Higher Morita Categories

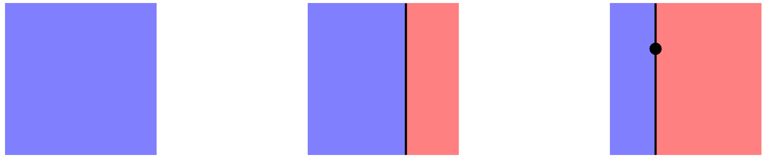
A fully extended (here: framed) topological field theory is a symmetric monoidal functor out of the (framed) bordism category. For the specific target category $\mathrm{Alg}_n(\mathcal{S})$, this becomes

$$\mathcal{Z}:\ \mathrm{Bord}_n^{\mathrm{fr}} \to \mathrm{Alg}_n(\mathcal{S})$$

Informally, the **higher Morita category** $\mathrm{Alg}_n(\mathcal{S})$ is

n-cat.	**standard version**	**factorization version**
0-morp.	E_n-algebra	locally constant on $(0,1)^n$
1-morp.	bimodule of E_n-algebras	constructible for $\{a^1\} \times (0,1)^{n-1}$
⋮	⋮	⋮
n-morp.	bimodule of ... bi-modules	constructible for full flags
(n+1)-morp.	bimodule map	map of factorization alge-bras
⋮	⋮	⋮

For $n=2$, the factorization version corresponds to *constructible factorization algebras* on stratified spaces like illustrated below



object 1-morphism 2-morphism

Theorem (Gwilliam-Scheimbauer, '18)**.** *Let $\mathcal{S}$ be a symmetric monoidal $\otimes$-sifted cocomplete $(\infty,1)$-category. The symmetric monoidal $(\infty,n+1)$-category $Alg_n(\mathcal{S})$ is fully n-dualizable.*

Conjecture (Lurie, '09)**.** *An E_n-algebra $\mathcal{A} \in Alg_n(\mathcal{S})$ is $(n+1)$- dualizable if, and only if, it is dualizable over the factorization homologies*

$$\int_{S^{k-1}\times\mathbb{R}^{n-k+1}} \mathcal{A}, \qquad \text{for } k = 0,1,\ldots,n.$$

This is proven for $n=1$ by Lurie, and for $n=2$ by Brochier-Jordan-Snyder.

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